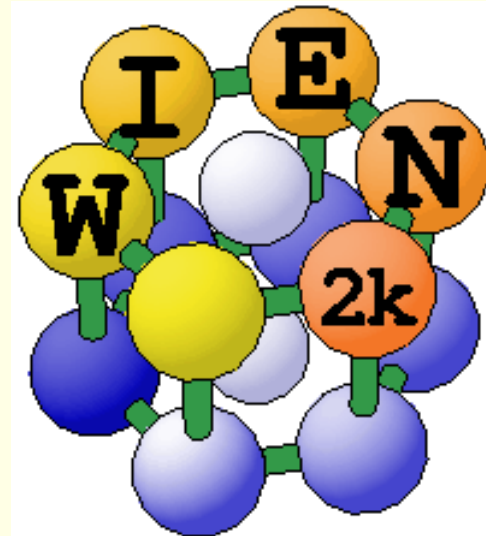


# Electronic structure, atomic forces and structural relaxations by WIEN2k

**Peter Blaha**

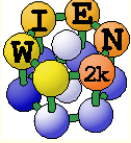
Institute of Materials Chemistry  
TU Vienna, Austria





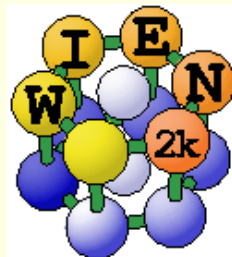
# Outline:

- APW-based methods (history and state-of-the-art)
- WIEN2k
  - *program structure + usage*
  - *forces, structure relaxation*
- Applications
  - *Phasetransitions in Aurivillius phases*
  - *Structure of Pyrochlore  $Y_2Nb_2O_7$*

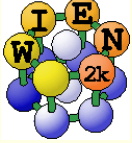


# APW based schemes

- **APW (J.C.Slater 1937)**
  - *Non-linear eigenvalue problem*
  - *Computationally very demanding*
- **LAPW (O.K.Anderssen 1975)**
  - *Generalized eigenvalue problem*
  - *Full-potential total-energies (A.Freeman et al.)*
- **Local orbitals (D.J.Singh 1991)**
  - *treatment of semi-core states (avoids ghostbands)*
- **APW+lo (E.Sjöstedt, L.Nordström, D.J.Singh 2000)**
  - *Efficiency of APW + convenience of LAPW*
  - *Basis for*

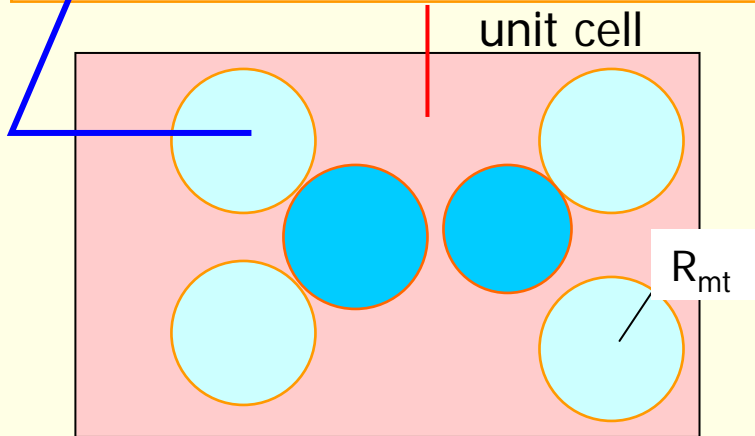


K.Schwarz, P.Blaho, G.K.H.Madsen,  
Comp.Phys.Commun. **147**, 71-76 (2002)

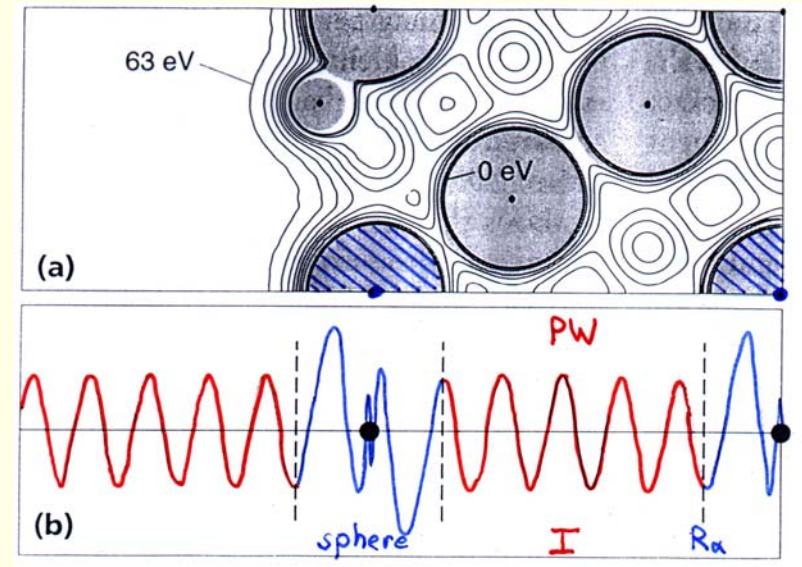


# APW Augmented Plane Wave method

The unit cell is partitioned into:  
atomic spheres  
Interstitial region



Bloch wave function:  
atomic partial waves  
Plane Waves (PWs)



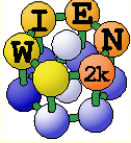
PW:  $e^{i(\vec{k} + \vec{K}) \cdot \vec{r}}$

join

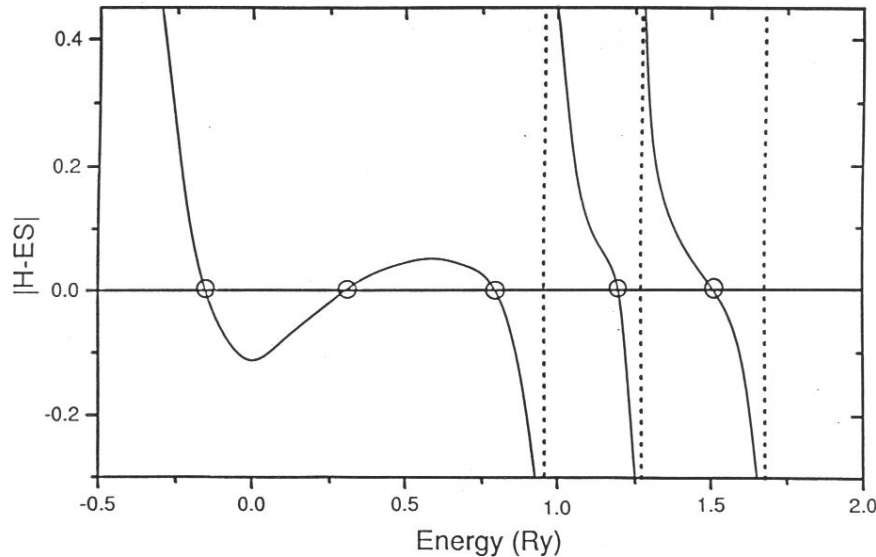
Atomic partial waves

$$\sum_{lm} a_{lm}^K u_l(r', \varepsilon) Y_{lm}(\hat{r}')$$

matching coefficient, radial function, spherical harmonics



# Slater's APW (1937)



Atomic partial waves

$$\sum_{\ell m} a_{\ell m}^K u_{\ell}(r', \varepsilon) Y_{\ell m}(\hat{r}')$$

Energy dependent

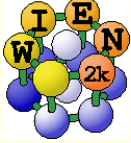
Radial basis functions lead to

H Hamiltonian  
S Overlap matrix

Non-linear eigenvalue problem

Computationally very demanding

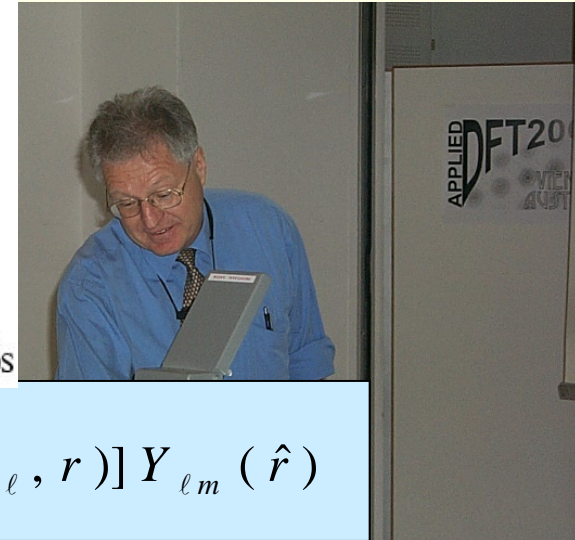
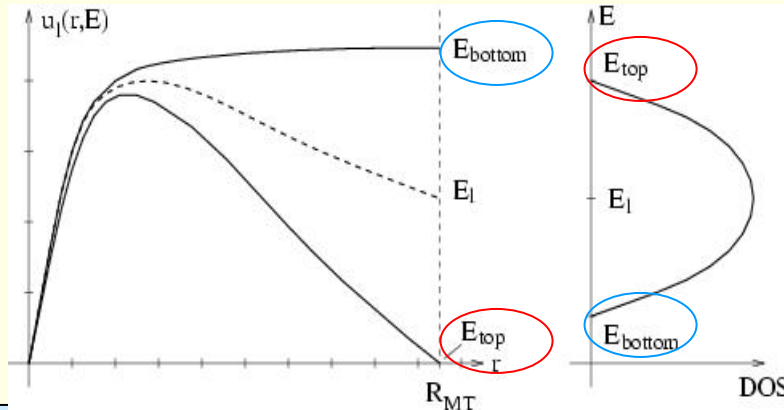
One had to numerically search for the energy, for which the  $\det|H-ES|$  vanishes.



# Linearization of energy dependence

**LAPW** suggested by

O.K.Andersen,  
Phys.Rev. B 12, 3060  
(1975)



$$\Phi_{k_n} = \sum_{\ell m} [A_{\ell m}(k_n)u_{\ell}(E_{\ell}, r) + B_{\ell m}(k_n)\dot{u}_{\ell}(E_{\ell}, r)]Y_{\ell m}(\hat{r})$$

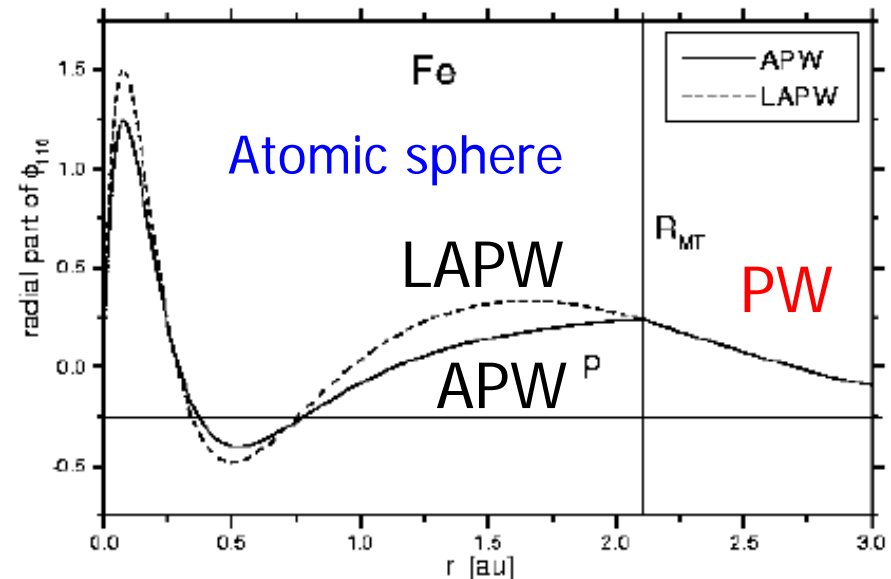
$$\dot{u}_l = \partial u_l / \partial \varepsilon$$

join PWs in value and slope

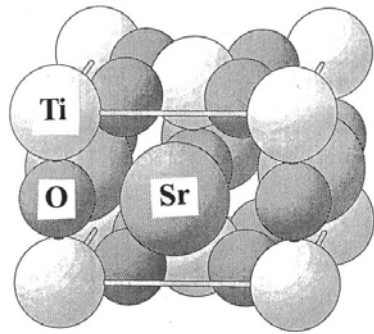
$u_l$  expanded at  $E_l$  („energy parameter“)

→ additional constraint requires more  
PWs than APW

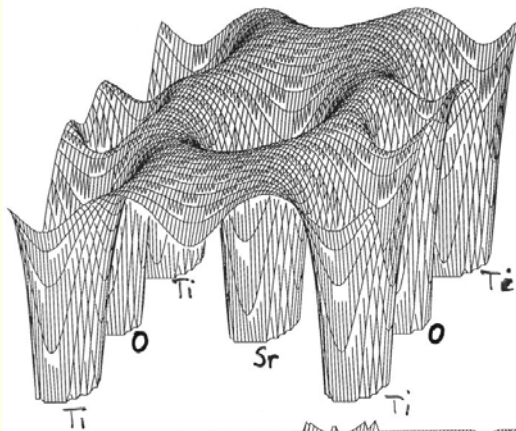
basis flexible enough for single  
diagonalization



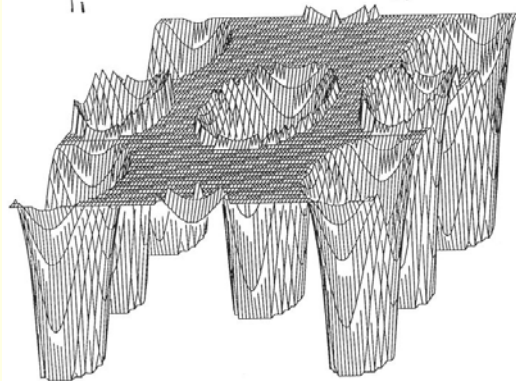
# Full-potential total-energy (A.Freeman et al.)



$\text{SrTiO}_3$



Full  
potential



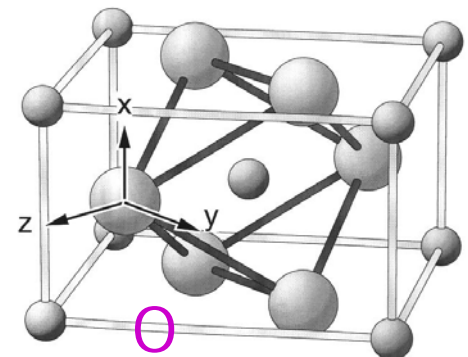
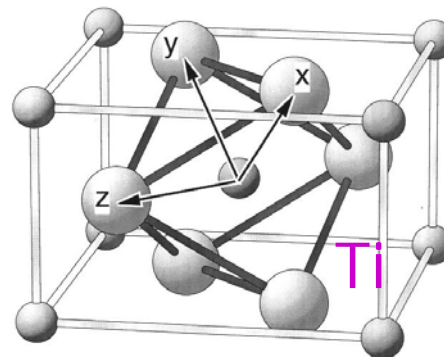
Muffin tin  
approximation

$\text{TiO}_2$  rutile

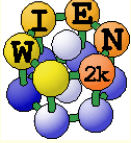
- The potential (and charge density) can be of general form (no shape approximation)

$$V(r) = \begin{cases} \sum_{LM} V_{LM}(r) Y_{LM}(\hat{r}) & r < R_a \\ \sum_K V_K e^{i\vec{K} \cdot \vec{r}} & r \in I \end{cases}$$

- Inside each atomic sphere a local coordinate system is used (defining LM)

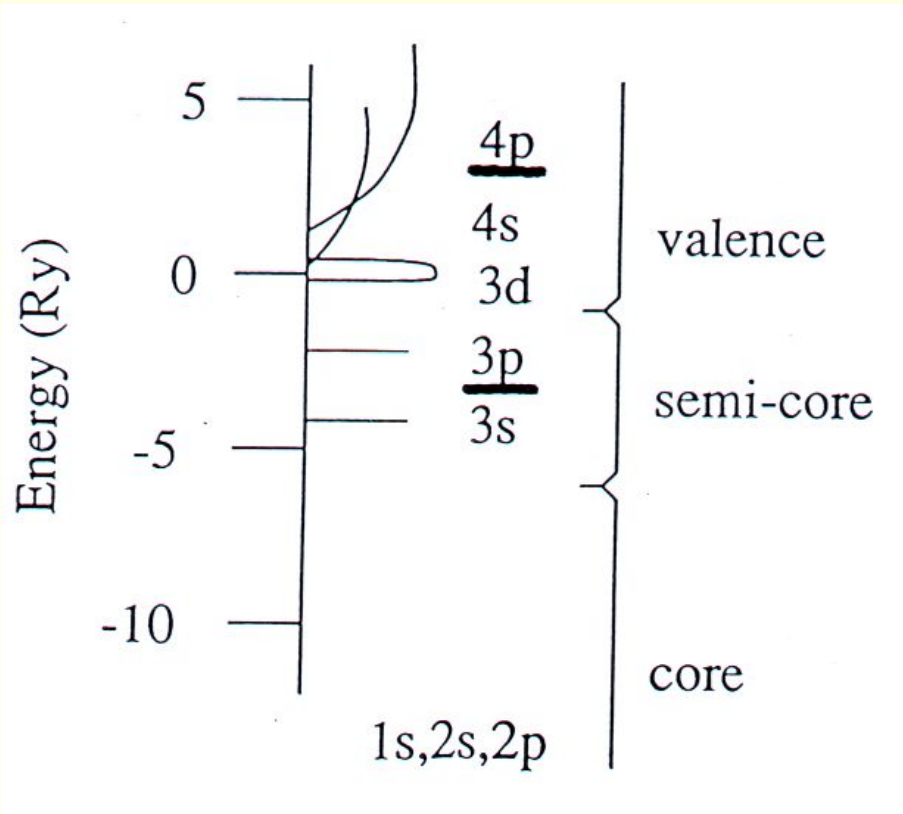






# Core, semi-core and valence states

For example: Ti



## ■ Valences states

- *High in energy*
- *Delocalized wavefunctions*

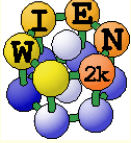
## ■ Semi-core states

- *Medium energy*
- *Principal QN one less than valence (e.g. in Ti 3p and 4p)*
- *not completely confined inside sphere*

## ■ Core states

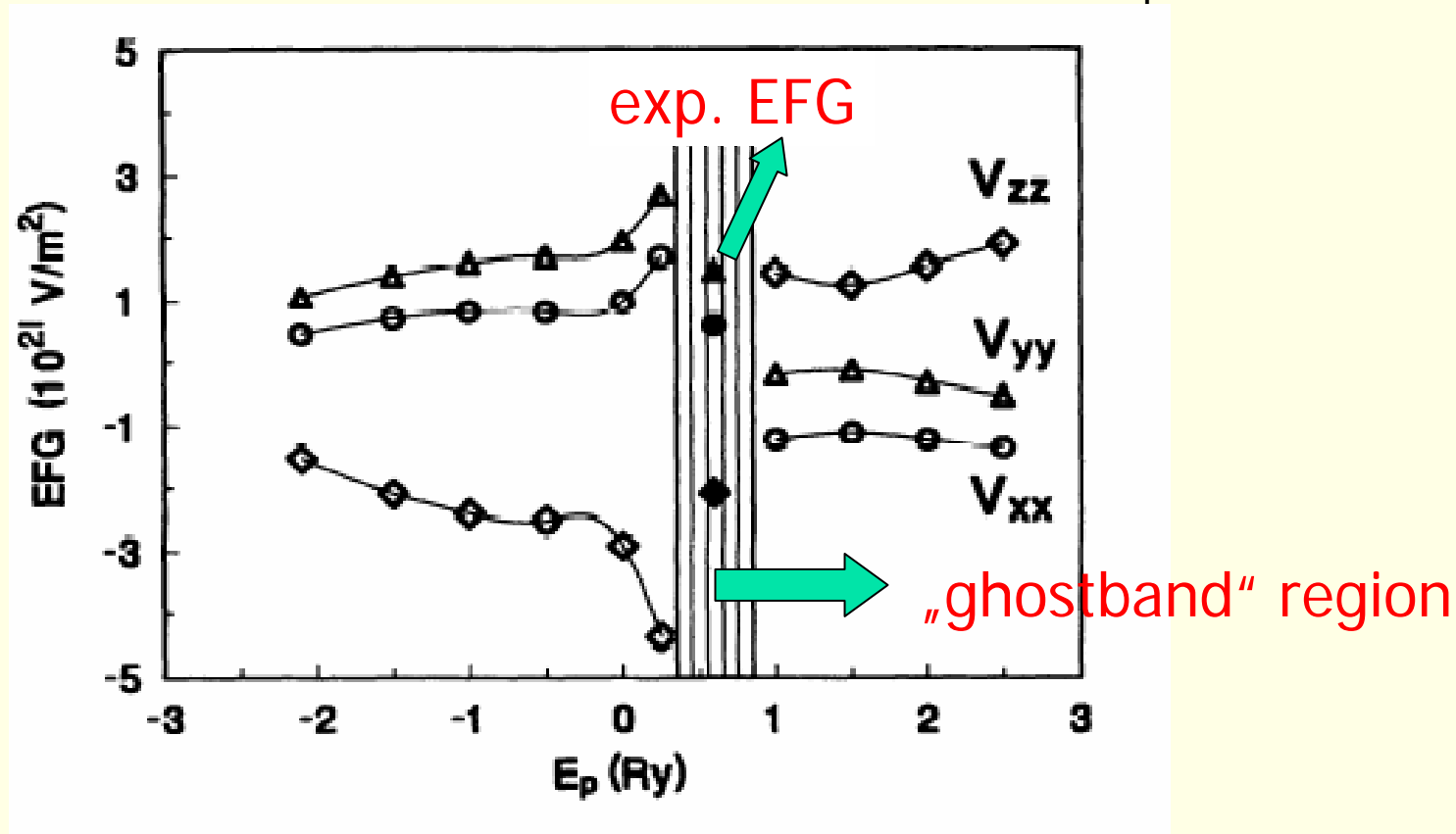
- *Low in energy*
- *Reside inside sphere*





# Problems of the LAPW method:

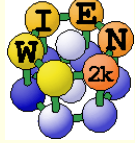
EFG Calculation for Rutile  $\text{TiO}_2$  as a function of the Ti- $p$  linearization energy  $E_p$



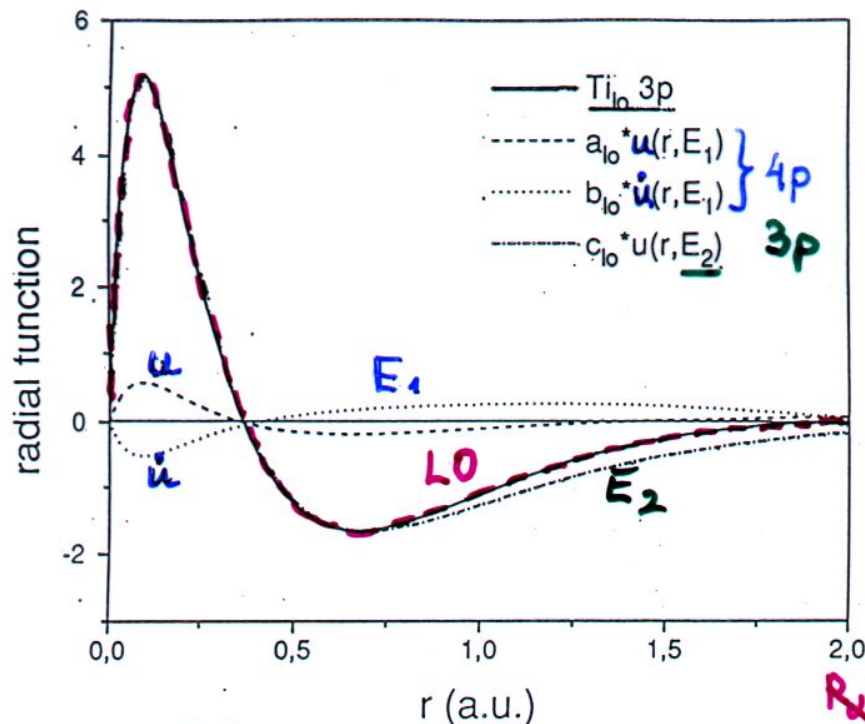
P. Blaha, D.J. Singh, P.I. Sorantin and K. Schwarz,  
Phys. Rev. B **46**, 1321 (1992).



# Problems of the LAPW method

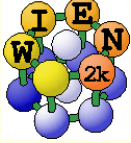
# Extending the basis: Local orbitals (LO)



$$\Phi_{LO} = [A_{lm}u_{\ell}^{E_1} + B_{lm}\dot{u}_{\ell}^{E_1} + C_{lm}u_{\ell}^{E_2}]Y_{\ell m}(\hat{r})$$

- LO
  - ↑ 4p
  - ↑ 3p
- *is confined to an atomic sphere*
- *has zero value and slope at R*
- *can treat two principal QN n for each azimuthal QN  $\ell$  (3p and 4p)*
- *corresponding states are strictly orthogonal (no "ghostbands")*
- *tail of semi-core states can be represented by plane waves*
- *only slight increase of basis set (matrix size)*

D.J.Singh,  
Phys.Rev. B 43 6388 (1991)



E.Sjöstedt, L.Nordström, D.J.Singh,  
*An alternative way of linearizing the augmented plane wave method,*  
Solid State Commun. 114, 15 (2000)

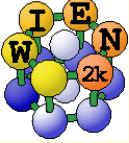
- Use **APW**, but at **fixed  $E$** , (superior PW convergence)
- **Linearize** with **additional lo** (add a few basis functions)

$$\Phi_{k_n} = \sum_{\ell m} A_{\ell m}(k_n) u_{\ell}(E_{\ell}, r) Y_{\ell m}(\hat{r})$$

$$\Phi_{lo} = [A_{\ell m} u_{\ell}^{E_1} + B_{\ell m} \dot{u}_{\ell}^{E_1}] Y_{\ell m}(\hat{r})$$

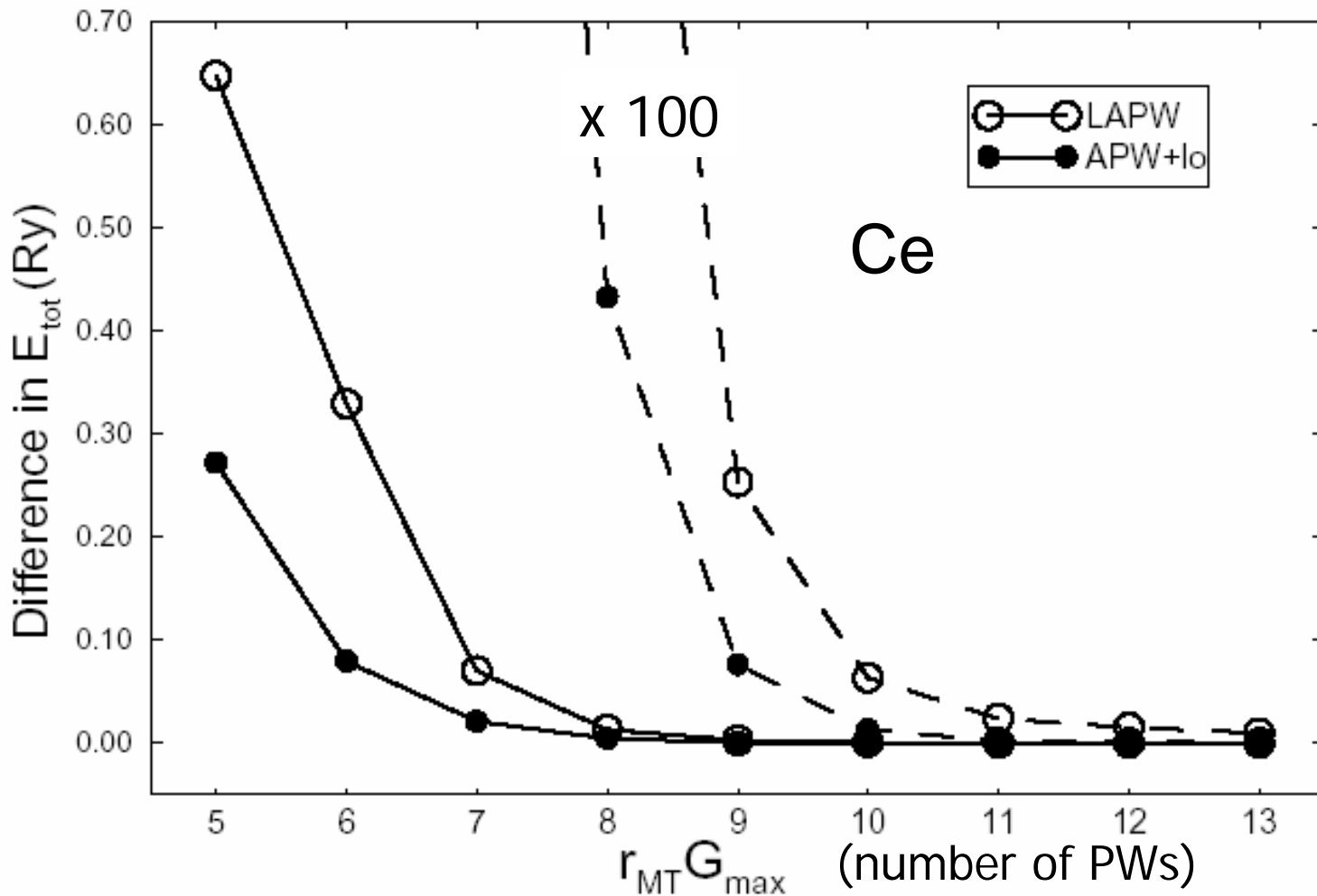
**optimal solution:** mixed basis (K.Schwarz, P.Blaha, G.K.H.Madsen,  
Comp.Phys.Commun. **147**, 71-76 (2002))

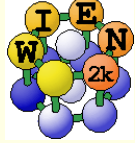
- use APW+lo for states which are difficult to converge:  
(f or d- states, atoms with small spheres)
- use LAPW+LO for all other atoms and angular momenta



# Convergence of the APW+LO Method

E. Sjöstedt, L. Nordström and D.J. Singh, Solid State Commun. **114**, 15 (2000).



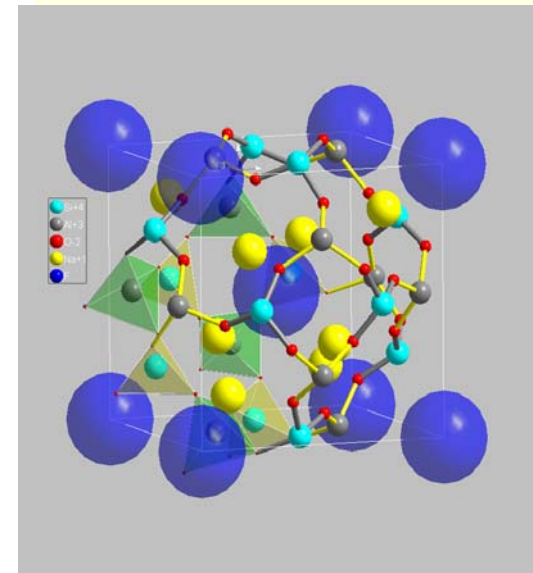
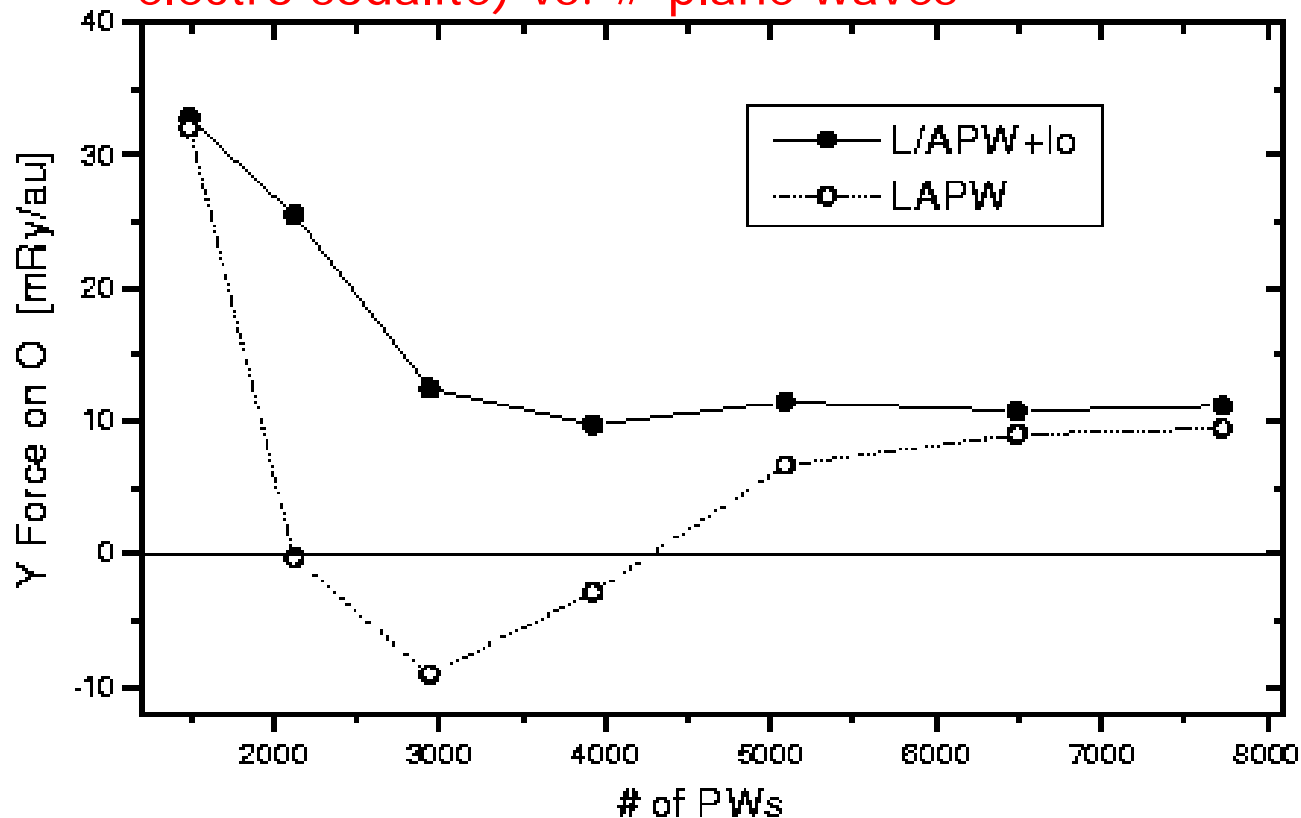


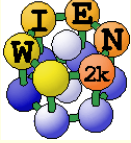
# Improved convergence of APW+lo

K.Schwarz, P.Blaha, G.K.H.Madsen,  
Comp.Phys.Commun. **147**, 71-76 (2002)

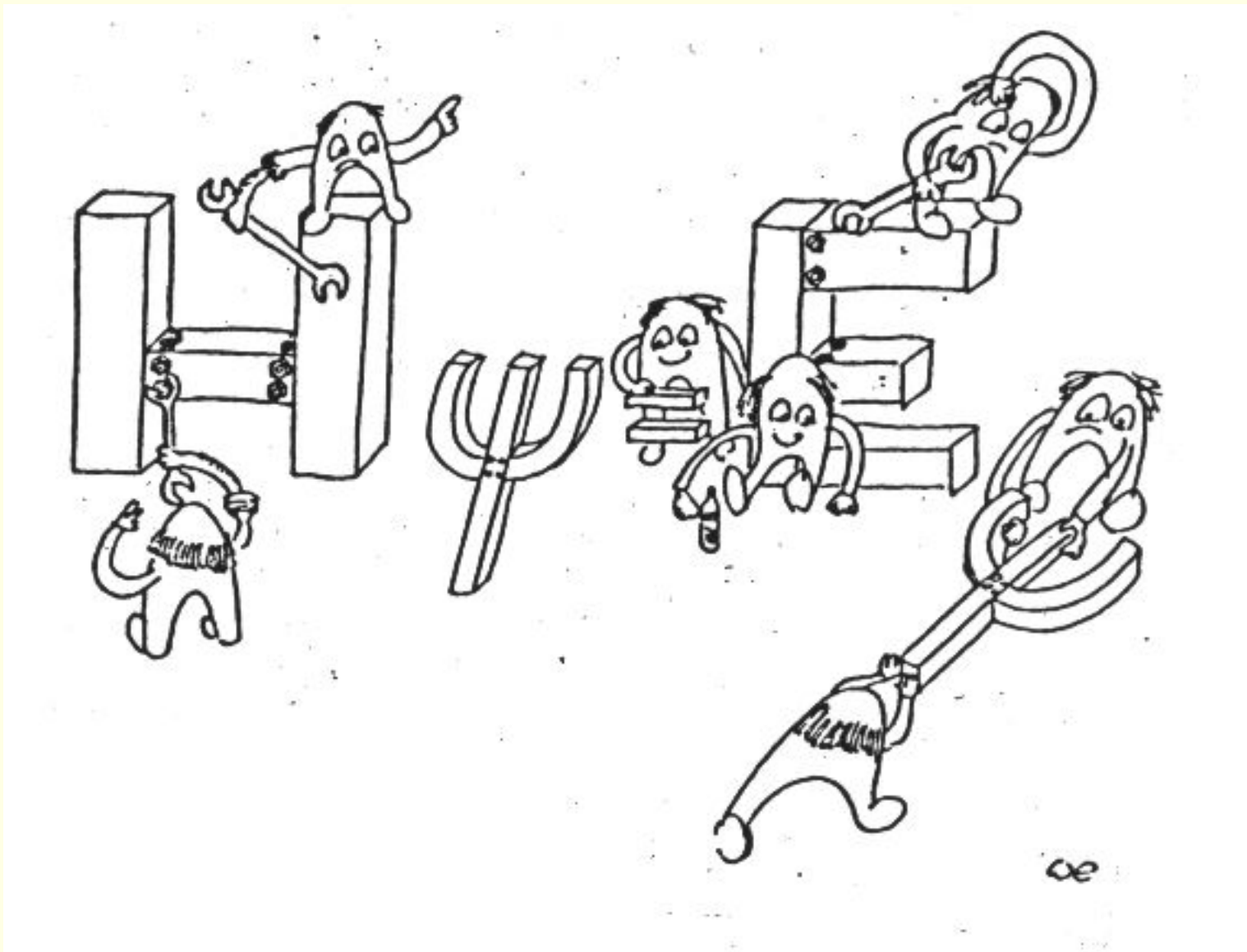
- changes sign and converges slowly in **LAPW**
- better convergence in **APW+lo**

Force ( $F_y$ ) on oxygen in SES (sodium electro sodalite) vs. # plane waves





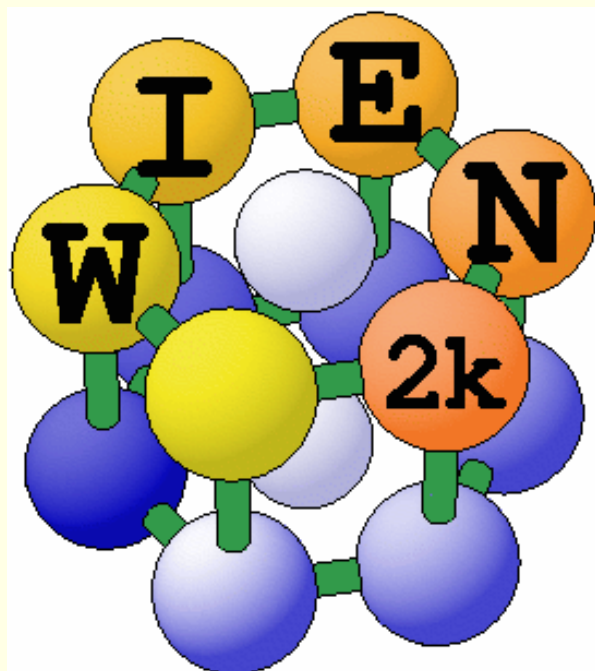
# Quantum mechanics at work







# WIEN2k software package



WIEN97: ~500 users  
WIEN2k: ~730 users

**An Augmented Plane Wave Plus Local  
Orbital  
Program for Calculating Crystal Properties**

**Peter Blaha  
Karlheinz Schwarz  
Georg Madsen  
Dieter Kvasnicka  
Joachim Luitz**

November 2001  
Vienna, AUSTRIA  
Vienna University of Technology

<http://www.wien2k.at>



# Development of WIEN2k



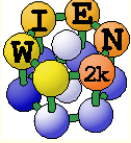
## ■ Authors of WIEN2k

*P. Blaha, K. Schwarz, D. Kvasnicka, G. Madsen and J. Luitz*

## ■ Other contributions to WIEN2k

- *C. Ambrosch-Draxl (Univ. Graz, Austria), optics*
- *U. Birkenheuer (Dresden), wave function plotting*
- *T. Charpin (Paris), elastic constants*
- *R. Dohmen und J. Pichlmeier (RZG, Garching), parallelization*
- *R. Laskowski (Vienna), non-collinear magnetism*
- *P. Novák and J. Kunes (Prague), LDA+U, SO*
- *B. Olejnik (Vienna), non-linear optics*
- *C. Persson (Uppsala), irreducible representations*
- *M. Scheffler (Fritz Haber Inst., Berlin), forces*
- *D.J.Singh (NRL, Washington D.C.), local orbitals (LO), APW+lo*
- *E. Sjöstedt and L Nordström (Uppsala, Sweden), APW+lo*
- *J. Sofo and J. Fuhr (Barriloché), Bader analysis*
- *B. Yanchitsky and A. Timoshevskii (Kiev), spacegroup*

## ■ and many others ....

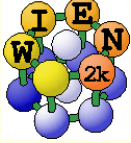


## ■ More than 700 user groups worldwide

- *40 industries* (Canon, Eastman, Exxon, Fuji, A.D.Little, Mitsubishi, Motorola, NEC, Norsk Hydro, Osram, Panasonic, Samsung, Sony, Sumitomo).
- *Europe:* (EHT Zürich, MPI Stuttgart, Dresden, FHI Berlin, DESY, TH Aachen, ESRF, Prague, Paris, Chalmers, Cambridge, Oxford)
- *America:* ARG, BZ, CDN, MX, USA (MIT, NIST, Berkeley, Princeton, Harvard, Argonne NL, Los Alamos Nat.Lab., Penn State, Georgia Tech, Lehigh, Chicago, SUNY, UC St.Barbara, Toronto)
- *far east:* AUS, China, India, JPN, Korea, Pakistan, Singapore, Taiwan (Beijing, Tokyo, Osaka, Sendai, Tsukuba, Hong Kong)

## ■ Registration at [www.wien2k.at](http://www.wien2k.at)

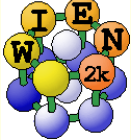
- 400/4000 Euro for Universities/Industries
- code download via *www* (with password), updates, bug fixes, news
- usersguide, *faq*-page, mailing-list with help-requests



# WIEN code as benchmark



- Full-potential nonorthogonal local-orbital minimum-basis band-structure scheme (FPLO)
  - Secular equation
  - Core-valence transformation
  - Site representation of density and potential
  - Basis optimization
- Comparison of results from FPLO and WIEN97
  - Example of band structure:  $\text{CaCuO}_2$
  - Total energies
  - Example of a semiconductor: diamond
- Summary

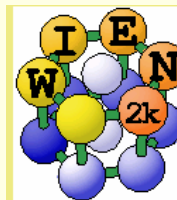


# WIEN2k- hardware/software



- WIEN2k runs on any **Unix/Linux** platform from PCs, workstations, clusters to supercomputers
- **Fortran90** (dynamical allocation)
- many individual **modules**, linked together with **C-shell** or perl-scripts
- **f90 compiler**, **BLAS-library** (mkl, Atlas), perl5, ghostview, gnuplot, Tcl/Tk (Xcrysden), www-browser

- web-based GUI – **w2web**
- **real/complex** version (inversion)
- 10 atom cells on **128Mb PC**
- 100 atom cells require **1-2 Gb RAM**
- **k-point parallel** on clusters with common NFS (**slow** network)
- **MPI/Scalapack** parallelization for big cases (>50 atoms) and **fast** network
- installation support for many platforms



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[StructGen™](#)  
[initialize calc.](#)  
[run SCF](#)  
[single program](#)  
[command line](#)

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[Tasks >>](#)

[Files >>](#)  
[struct file\(s\)](#)  
[input files](#)  
[output files](#)  
[SCF files](#)

[Session Mgmt. >>](#)  
[change session](#)  
[change directory](#)  
[change info](#)

[Configuration >>](#)

[Usersguide](#)  
[html-Version](#)  
[pdf-Version](#)

Session: **TiC** User: **pblaha**  
/susi/pblaha/lapw/TiC

Thu Nov 29 13:22:25 2001  
STATUS: idle  
[refresh](#) [norefresh](#)

**w2web**, the fully web-enabled interface to WIEN2k

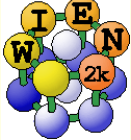
Session Name:	TiC
Session ID:	599582
Directory:	/susi/pblaha/lapw/TiC
Last changed:	Wed Nov 21 10:12:17 2001
Comments:	

- ☐ spin polarized calculation
- ☐ AFM calculation
- ☐ complex calculation (no inversion)
- ☐ parallel calculation

Change session information



w2web © luitz.at



# W2web – A web-interface

## ■ Based on www

- *WIEN2k can be managed remotely via w2web*
- *Secure (Password)*

## ■ Important steps:

- *start w2web on all your hosts*
- *use your favorite browser and connect to the master host:port*
- *define a session on the desired host*

Welcome to *w2web*

the fully web-enabled interface to WIEN2k

Select stored session:

show only selection

Fccni

**TiC**

TiO2

bccw

fccu\_ldau

fccu

fccyb\_sp

fccyb

he\_surf\_0

nibnn

si\_goessl

test

tic\_goessl

tic\_puls

tio\_puls

Select

Create new session:

Create

on host-node

**master node**

http://jupiter:10000

http://homer:10000

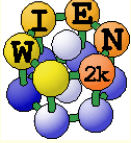
http://pauli:10000

http://fp98.zserv:10000

http://hal.zserv:10000

edit hosts





# GUI (Graphical user interface)

- **Structure generator**
  - *cif-file converter*
  - *spacegroup selection*
- **step by step initialization**
  - *symmetry detection*
  - *automatic input generation*
- **SCF calculations**
  - *Magnetism (spin-polarization)*
  - *Spin-orbit coupling*
  - *Forces (automatic geometry optimization)*
- **Guided Tasks**
  - *Energy band structure*
  - *DOS*
  - *Electron density*
  - *X-ray spectra*
  - *Optics*

Session: TIC User: pblaha /susi/pblaha/lapw/TiC

The Nov 23 13:23:41 2001  
STA/TUS:141  
[refresh](#) [home#web](#)

StructEdit /susi/pblaha/lapw/TiC

### StructGen™

You have to click "Save Structure" for changes to take effect!

[Save Structure](#)

Title:

Treatment of core: ☐ relativistic

Lattice:  
Type: F

inequivalent Atoms: 2

Lattice parameters in  ☐

a=  b=  c=   
a=  b=  g=

Atom 1:   Z=  [remove atom](#)

NPT=  RMT=  RO=  Non-cubic ☐

Pos 1: x=  y=  z=  [remove](#)  
[add position](#)

Atom 2:   Z=  [remove atom](#)

NPT=  RMT=  RO=  Non-cubic ☐

Pos 1: x=  y=  z=  [remove](#)  
[add position](#)

[add an atom](#)

Number of symmetry operations:  ☐

Execution >>  
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Utils. >>

Tasks >>

Files >>  
[struct file\(s\)](#)  
[input files](#)  
[output files](#)  
[SCF files](#)

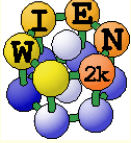
Session Mgmt. >>  
[change session](#)  
[change directory](#)  
[change info](#)

Configuration >>

Usersguide  
[html-Version](#)  
[pdf-Version](#)

Ides and realization by [hufsch](#) © 2001





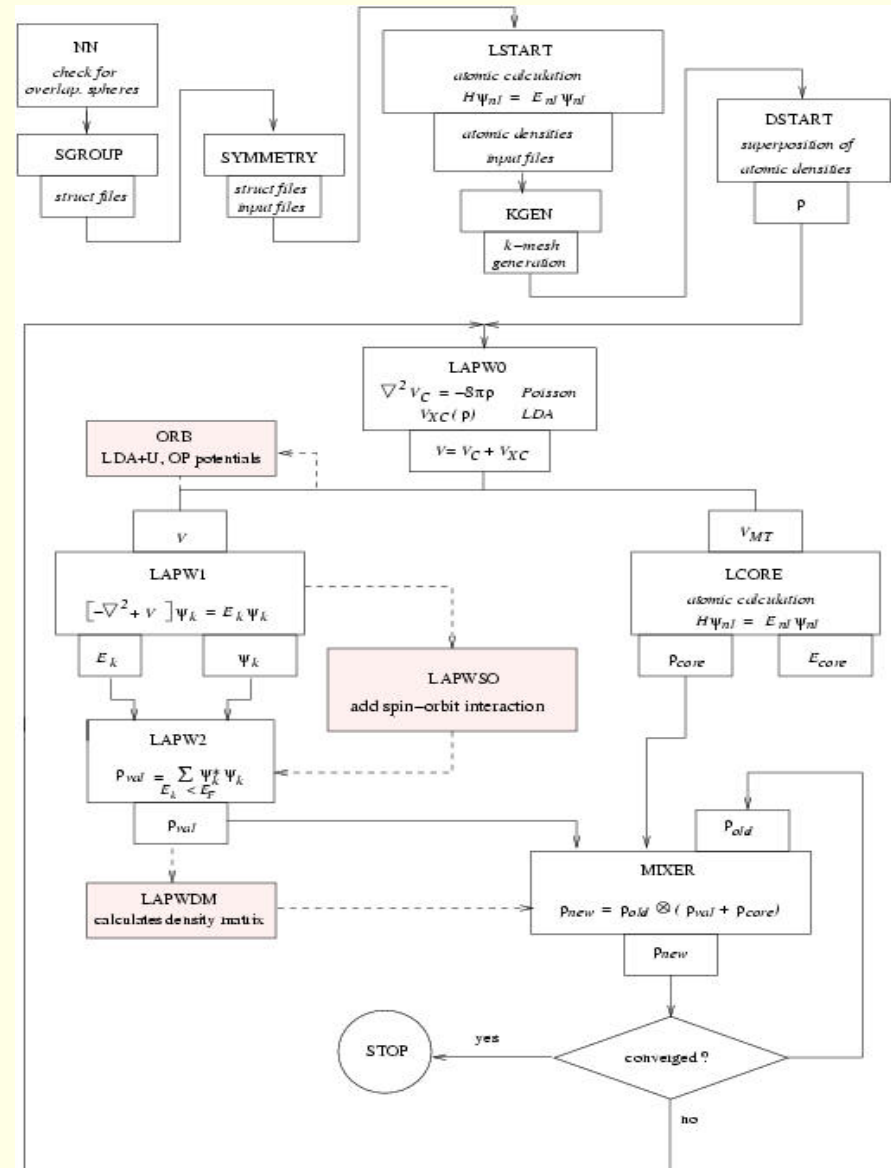
# Program structure of WIEN2k

## ■ init\_lapw

- *initialization*
- *symmetry detection (F, I, C-centering, inversion)*
- *input generation with recommended defaults*
- *quality (and computing time) depends on k-mesh and R.Kmax (determines #PW)*

## ■ run\_lapw

- *scf-cycle*
- *optional with SO and/or LDA+U*
- *different convergence criteria (energy, charge, forces)*





# Task for electron density plot

- A task consists of
  - *a series of steps*
  - *that must be executed*
  - *to generate a plot*
- For electron density plot
  - *select states (e.g. valence  $e^-$ )*
  - *select plane for plot*
  - *generate 3D or contour plot with gnuplot or Xcrysden*



Execution >>  
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[initialize calc.](#)  
[run SCF](#)  
[single program](#)  
[command line](#)

Utils. >>

<< Tasks  
[El. Dens.](#)  
[DOS](#)  
[XSPEC](#)  
[ELNES](#)  
[OPTIC](#)  
[Bandstructure](#)

Files >>  
[struct file\(s\)](#)  
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Session Mgmt. >>  
[change session](#)  
[change directory](#)  
[change info](#)

Configuration >>

Usersguide  
[html-Version](#)  
[pdf-Version](#)

Session: TiC User: pblaha  
/sus/pblaha/lapw/TiC

Thu Nov 23 13:26:30 2001  
STATUS: idle  
[refresh](#) [refresh](#)

## Electron density plots

edit TiC.in2 change EMIN

x lapw2 Calculate clmval

Calculate density with XCrysden

edit TiC.in5 Edit input-file

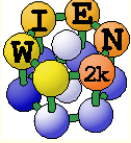
x lapw5 Calculate partial DOS

Preview density with XCrysden

rhplot Plot DOS

edit TiC.in2 reset EMIN





## ■ Energy bands

- *classification of irreducible representations*
- *‘character-plot’ (emphasize a certain band-character)*

## ■ Density of states

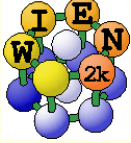
- *including partial DOS with l and m- character (eg.  $p_x$ ,  $p_y$ ,  $p_z$ )*

## ■ Electron density, potential

- *total-, valence-, difference-, spin-densities,  $\rho$  of selected states*
- *1-D, 2D- and 3D-plots (Xcrysden)*
- *X-ray structure factors*
- *Bader’s atom-in-molecule analysis, critical-points, atomic basins and charges ( $\nabla \rho \cdot \vec{n} = 0$ )*
- *spin+orbital magnetic moments (spin-orbit / LDA+U)*

## ■ Hyperfine parameters

- *hyperfine fields (contact + dipolar + orbital contribution)*
- *Isomer shift*
- *Electric field gradients*

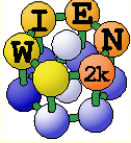


## ■ Spectroscopy

- *core levels (with core holes)*
- *X-ray emission, absorption, electron-energy-loss*
  - (core - valence/conduction-band transitions including matrix elements and angular dep.)
  - EELS inclusion of possible non-dipol transitions (momentum transfer)
- *optical properties (dielectric function, JDOS including momentum matrix elements and Kramers-Kronig)*
- *fermi surface (2D, 3D)*

## ■ Total energy and forces

- *optimization of internal coordinates, (MD, BROYDEN)*
- *cell parameter only via  $E_{tot}$  (no stress tensor)*
- *elastic constants for cubic cells*
- *Phonons via supercells*
  - interface to PHONON (K.Parlinski) – bands, DOS, thermodynamics, neutrons



# Total energies and atomic forces

(Yu et al.; Kohler et al.)

## ■ Total Energy:

- *Electrostatic energy*
- *Kinetic energy*
- *XC-energy*

$$U[\rho] = \frac{1}{2} \int d^3\vec{r} \rho(\vec{r}) V_{es}(\vec{r}) + \frac{1}{2} \sum_{\alpha} Z_{\alpha} V_{es}^{\alpha}(\vec{r})$$

$$T[\rho] = \sum_i n_i \varepsilon_i - \int d^3\vec{r} \rho(\vec{r}) V_{eff}(\vec{r})$$

$$E_{xc}[\rho] = \int d^3\vec{r} \rho(\vec{r}) \varepsilon_{xc}(\vec{r})$$

## ■ Force on atom $\alpha$ :

$$\vec{F}^{\alpha} = \frac{-dE_{tot}}{d\vec{R}_{\alpha}} = F_{HF}^{\alpha} + F_{core}^{\alpha} + F_{val}^{\alpha}$$

### ■ *Hellmann-Feynman-force*

$$F_{HF}^{\alpha} = Z_{\alpha} \sum_{m=-1}^1 \lim_{r_{\alpha} \rightarrow 0} \frac{V_{1m}^{es}(r_{\alpha})}{r_{\alpha}} \nabla_{\alpha} [r_{\alpha} Y_{1m}(\hat{r})]$$

### ■ *Pulay corrections*

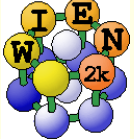
- *Core*
- *Valence*

$$F_{core}^{\alpha} = - \int \rho_{core}(r) \nabla_{\alpha} V_{eff}(r) d\vec{r}$$

- **expensive, contains a summation of matrix elements over all occupied states** (done only in last scf-iteration)

$$F_{val}^{\alpha} = \int V_{eff}(r) \nabla_{\alpha} \rho_{val}(r) d\vec{r} + \sum_{k,i} n_i \sum_{K,K'} c_i^{*}(K') c_i(K) \times$$

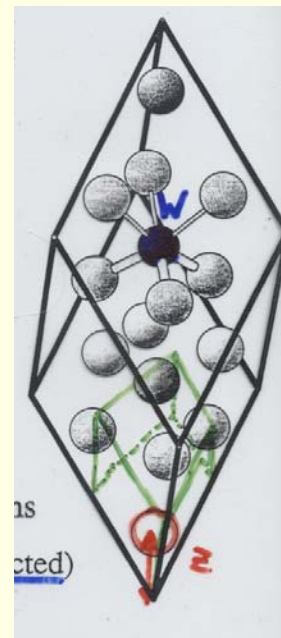
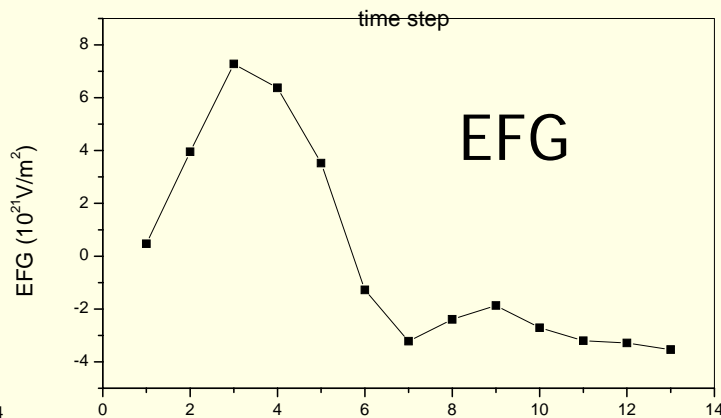
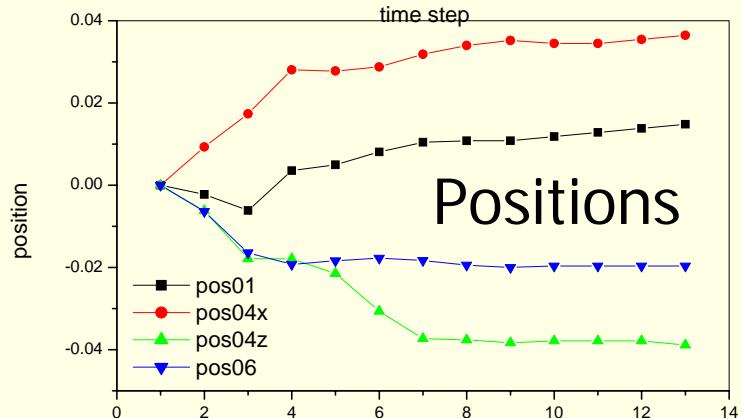
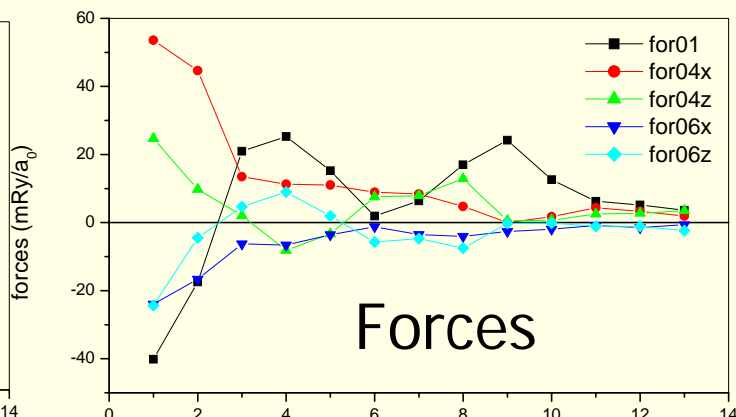
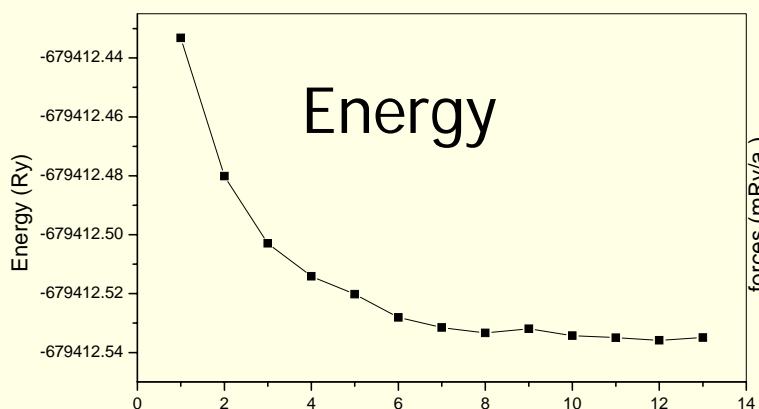
$$\left[ (K^2 - \varepsilon_i) \oint \phi_{K'}^{*}(r) \phi_K(r) dS_{\alpha} - i(K - K') \langle \phi_{K'} | H - \varepsilon_i | \phi_K \rangle_{\alpha} \right]$$



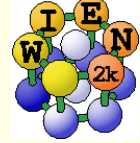
# Optimization of atomic positions (E-minimization via forces)

- damped Newton scheme  
(not user friendly, input adjustment by hand)
- **new quite efficient Broyden scheme**

W impurity in Bi (2x2x2 cell:  $\text{Bi}_{15}\text{W}$ )



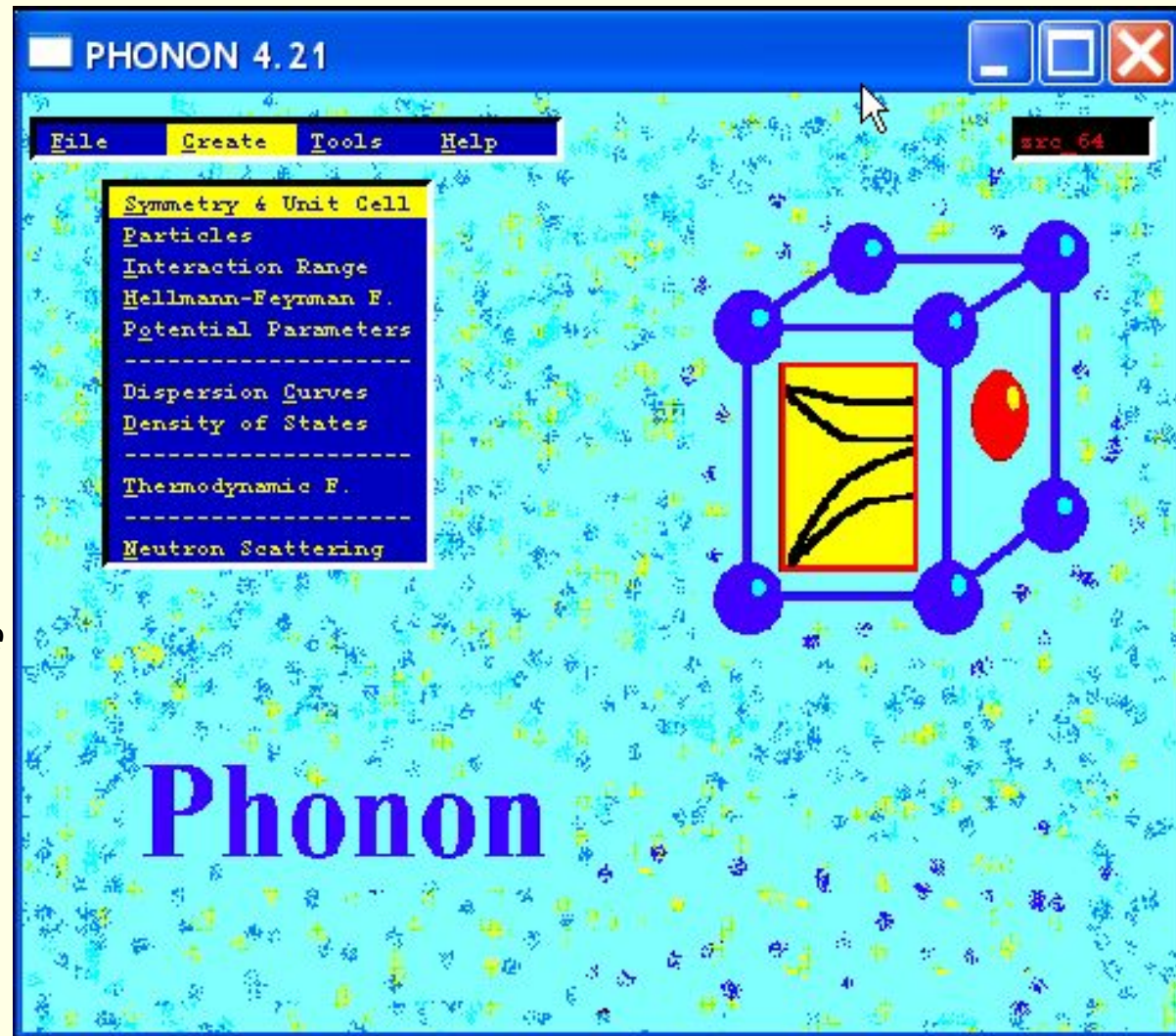




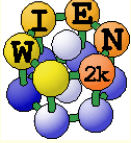
## ■ PHONON

- *by K. Parlinski (Crakow)*
- *runs under MS-windows*
- *uses a „direct“ method to calculate **Force-constants** with the help of an *ab initio* program*
- *with these Force-constants phonons at arbitrary *k*-points can be obtained*

- Define your spacegroup
- Define all atoms at **previously optimized positions**



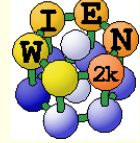




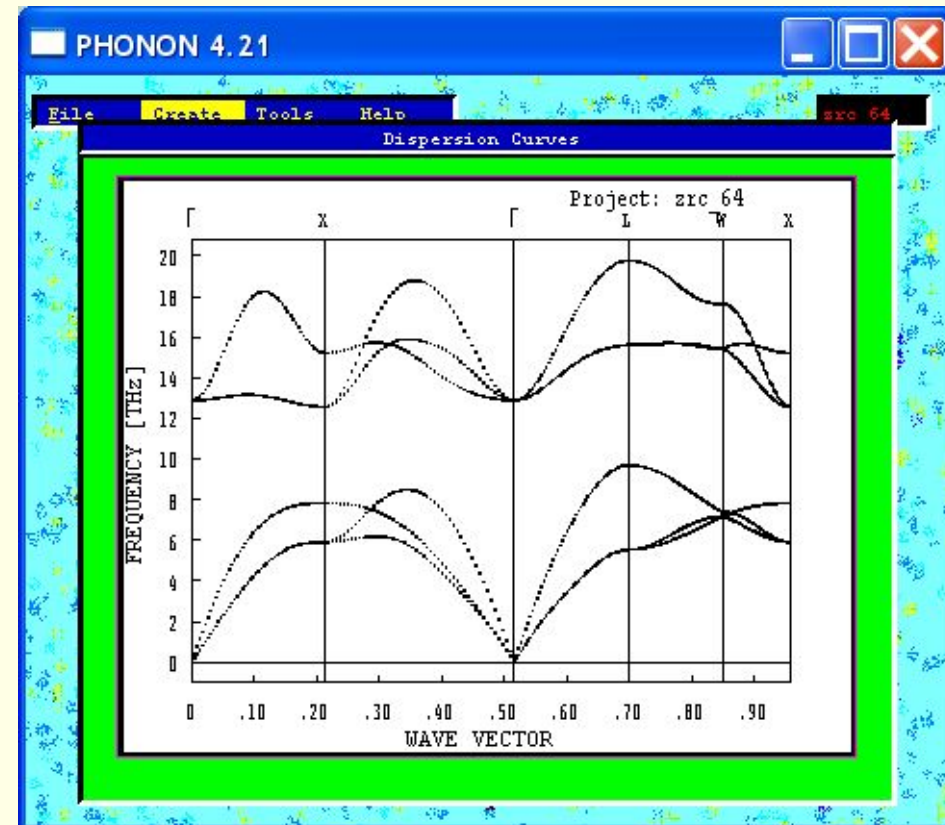
- Define an interaction range (supercell)
  - create *displacement* file
  - transfer *case.d45* to Unix
- Calculate forces for all required displacements
  - *init\_phonon\_lapw*
    - for each displacement a *case\_XX.struct* file is generated in an extra directory
    - runs *nn* and lets you define *RMT* values like:
      - 1.85 1-16

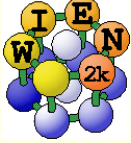


- *init\_lapw*: either *without symmetry* (and then copies this setup to all *case\_XX*) or *with symmetry* (must run *init\_lapw* for all *case\_XX*) (Do NOT use *SGROUP*)
- *run\_phonon*: *run\_lapw -fc 0.1 -i 40* for each *case\_XX*



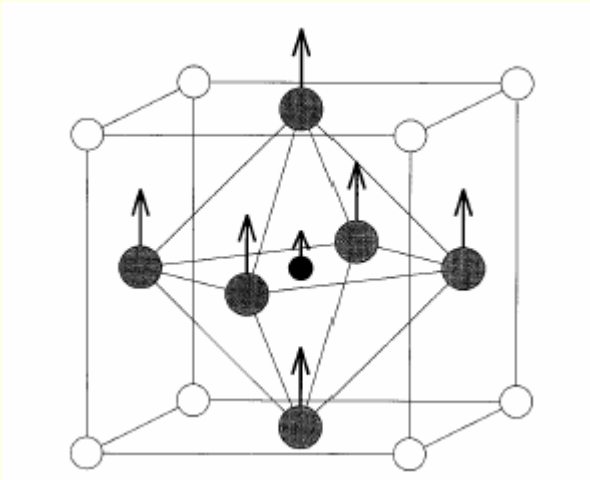
- **analyze\_phonon\_lapw**
  - reads the *forces* of the *scf* runs
  - generates „*Hellman-Feynman*“ file *case.dat* and a „symmetrized HF-file *case.dsy* (when you have displacements in both directions)
    - check quality of forces:
    - $\sum F_x$  should be small (0)
    - $\text{abs}(F_x)$  should be similar for  $\pm$ -displacements
- transfer *case.dat* (*dsy*) to Windows
- Import HF files to PHONON
- Calculate phonons





# Ferroelectricity in Aurivillius phases

- **\* Bilbao: J. Manuel Perez-Mato, M. Aroyo**  
Universidad del País Vasco
- **\* Vienna: P. Blaha, K. Schwarz,**  
J. Schweifer
- **\* Cracow: K. Parlinski**



**Classical Example of a Ferroelectric**

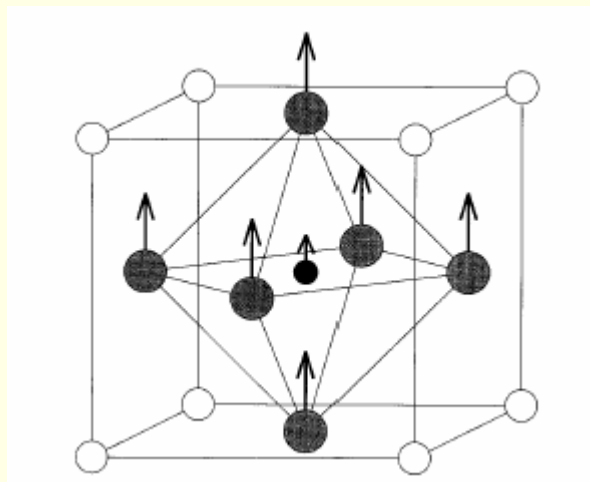
**$\text{PbTiO}_3$**

**(perovskite distorted structure)**

**Features:**

- spontaneous polarization- frozen polar mode
- phase transition:

paraelectric  $\xrightarrow{\text{T}_c}$  ferroelectric



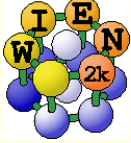
≈

- Symmetry break at T<sub>c</sub>:

cubic ---- tetragonal  
**Pm-3m ----- P4mm**

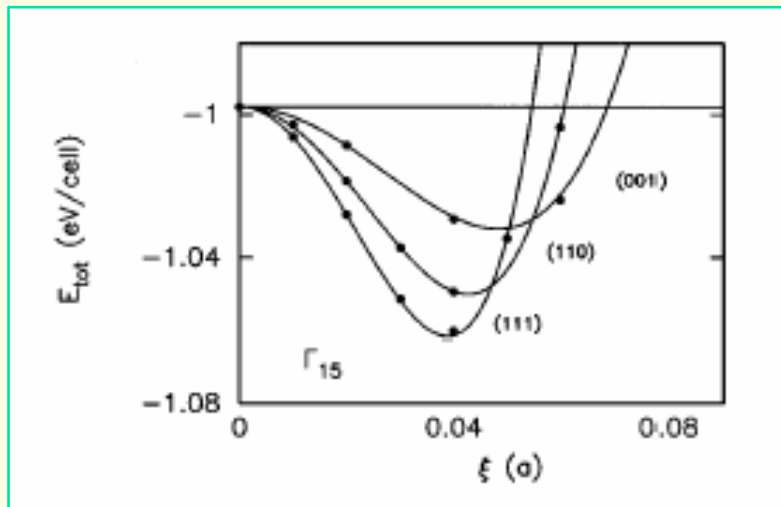
- A single (degenerate) *normal* mode responsible for the transition
- Structure in Ferroelectric Phase:  
**high-symmetry structure + frozen polar mode**

- Strain: spontaneous strain  $\Delta\epsilon \sim 10^{-2}$

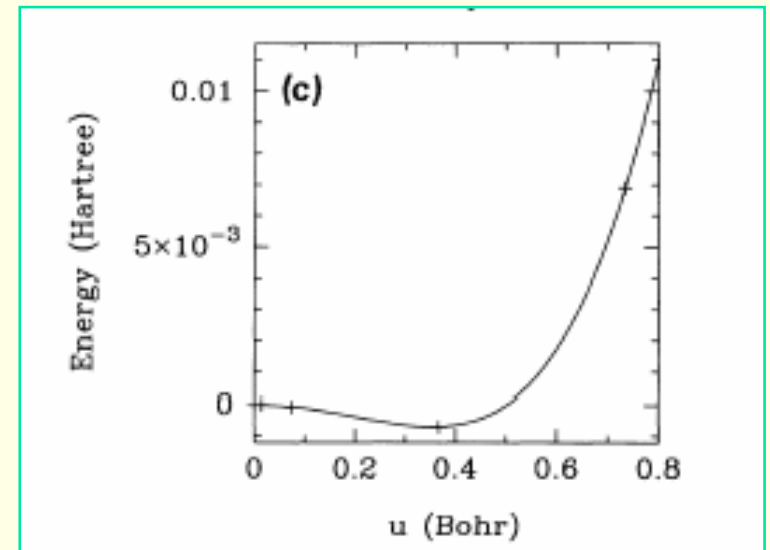


# Ab-initio “prediction” of the ferroelectric instability

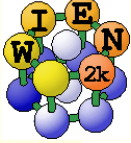
## $\text{PbTiO}_3$



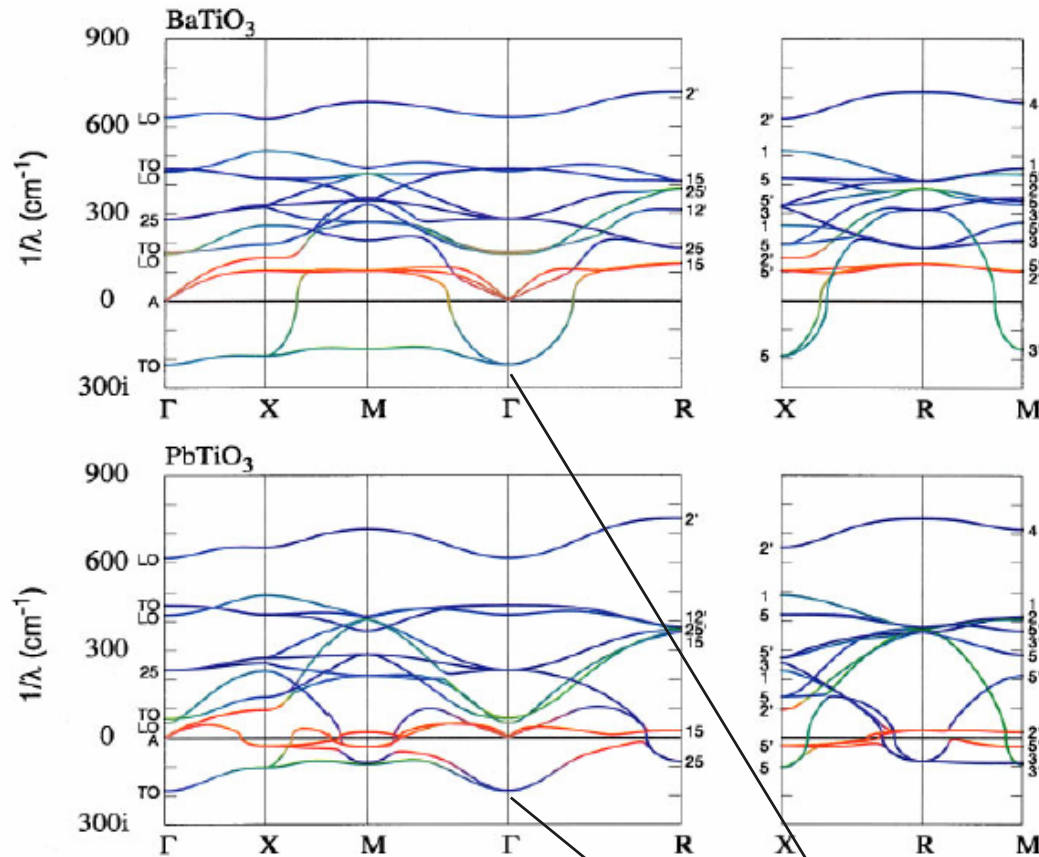
Waghmare&Rabe 1997



King-Smith&Vanderbilt 1994



## Ab-initio Phonon Branches



Ghosez et al. 1999

Polar unstable mode





**orthorhombic.**

## A2<sub>1</sub>am

## B2ab

## A2<sub>1</sub>am



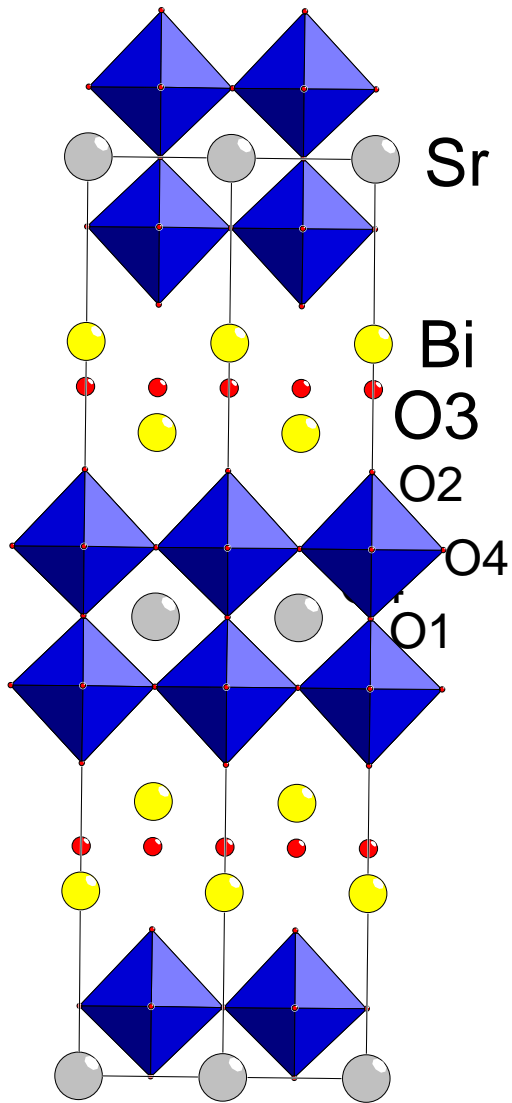
- **n-sized perovskite slab**

## cation deficient perovskites:

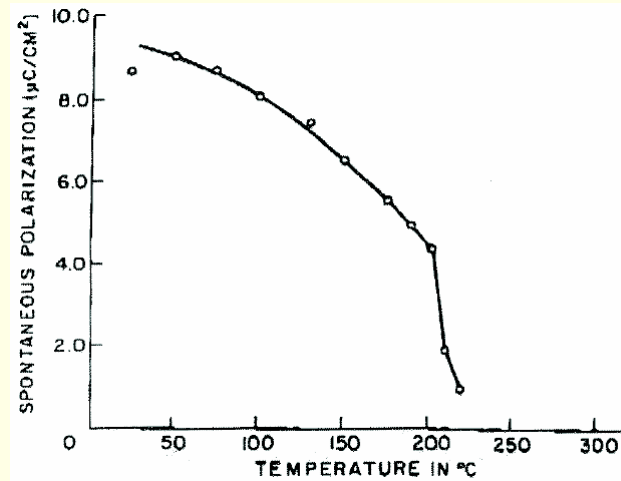


Ps

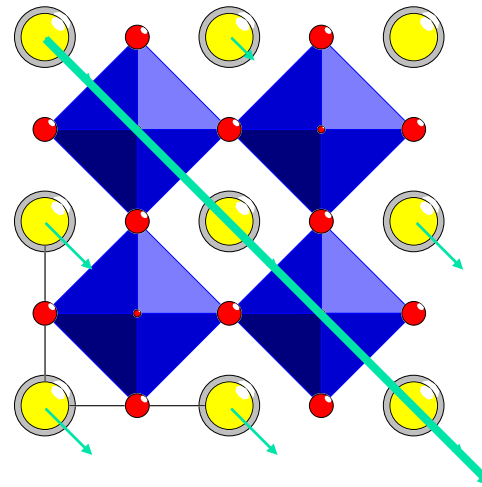
**Dipl. Thesis -S. Borg 2001**



## Spontaneous Polarization

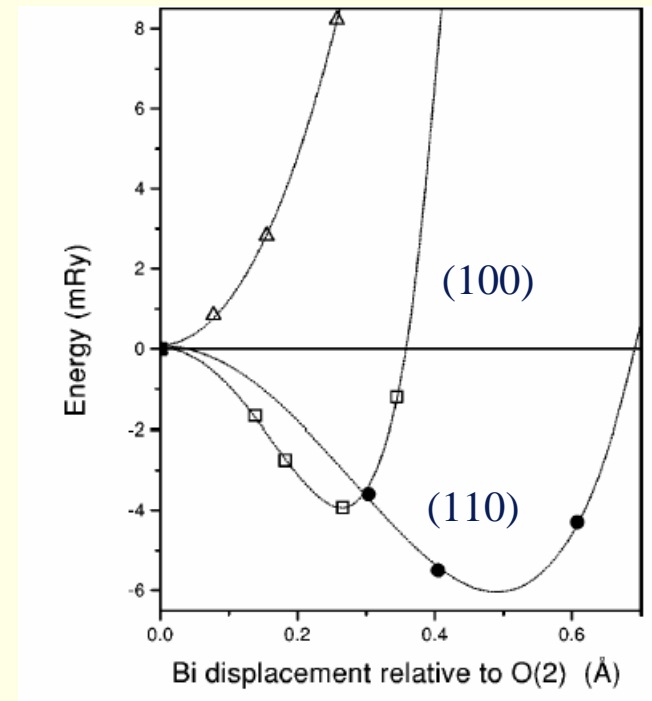
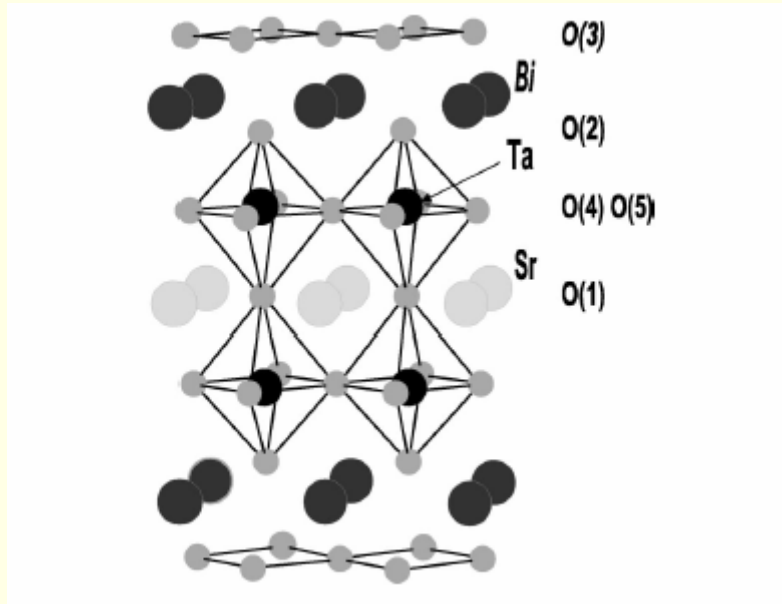


## polar mode Eu (deg. 2)

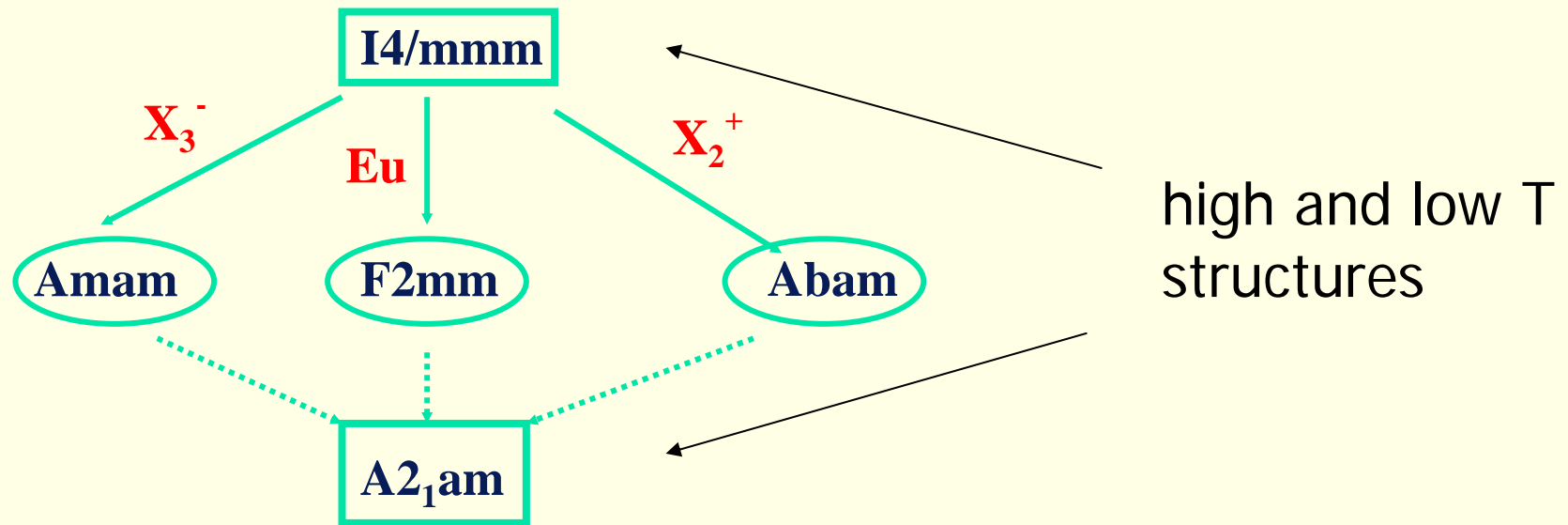
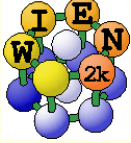


# Previous ab-initio calculations in SBT

Stachiotti et al. 2000



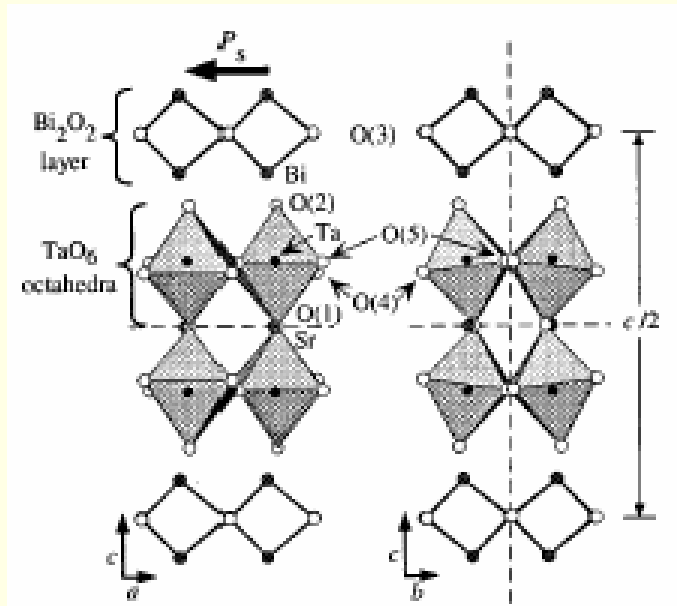
- **Unstable Eu polar mode**
- **Strong contribution of Bi displacement :  
Bi-O(2) hybridization**



From symmetry analysis (group/ subgroup relations) the  $Eu$  mode alone cannot explain the complete phase transition

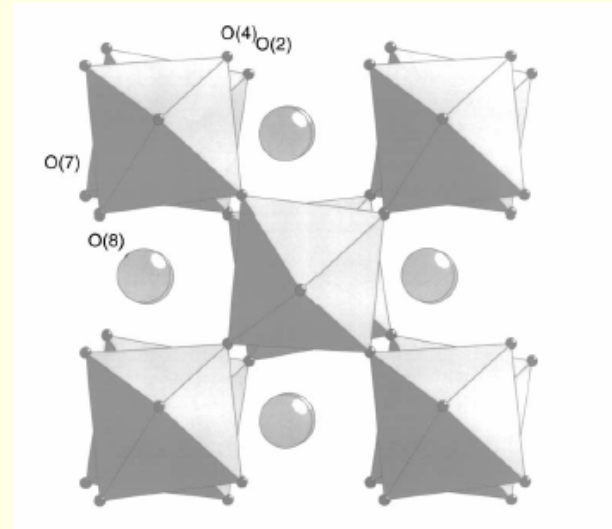
Ferroelectric Phase = mode  $Eu$  + mode  $X_3^-$  + mode  $X_2^+$

mode  $X_3^-$



Shimakawa et al. 1999

mode  $X_2^+$



Hervoches et al. 2002

Ferroelectric Phase = mode Eu + mode  $X_3^-$  + mode  $X_2^+$

Eigenvectors:

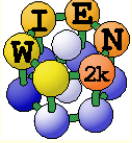
$e_{Eu}$

$e_{X3}$

$e_{X2}$

Mode Amplitudes (bohrs) :

$Q_{Eu} = 0.82$      $Q_{X3} = -1.20$      $Q_{X2} = -0.35$



# All Aurivillius: “Atypical” Ferroelectrics:

Prototype tetragonal paraelectric phase

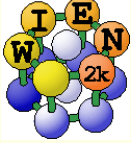
Not possible through the freezing  
of a polar mode!

Ferroelectric orthorhombic phase

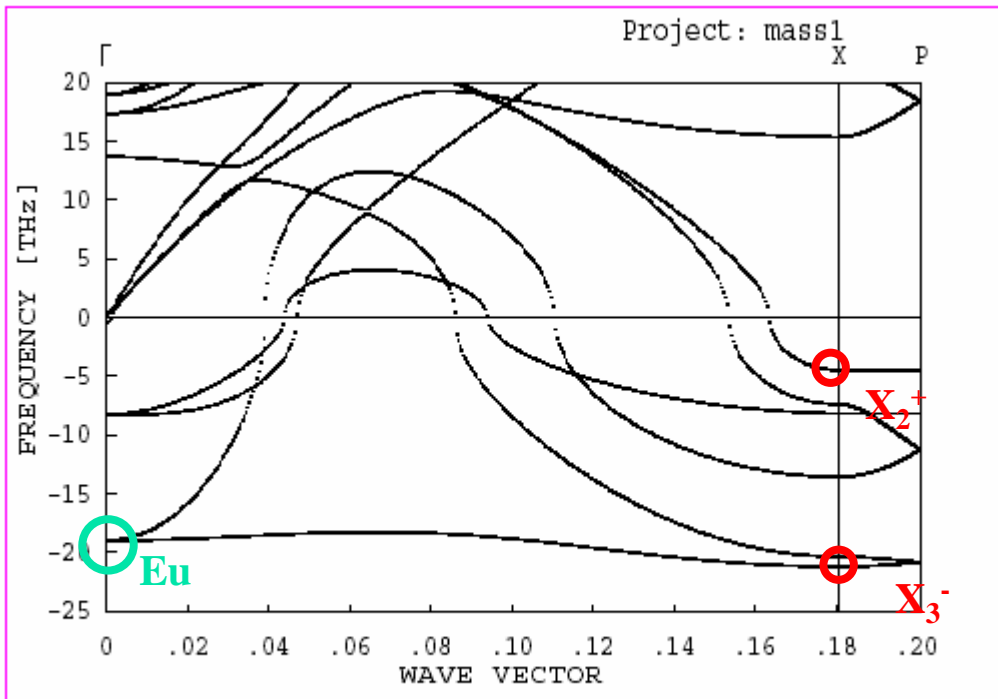
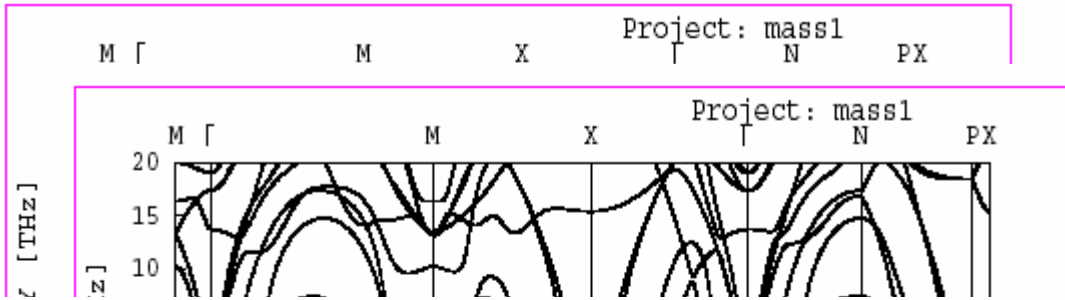
- A single order parameter can not produce this transition.

According to Landau theory:

- Either discontinuous transition or intermediate phases

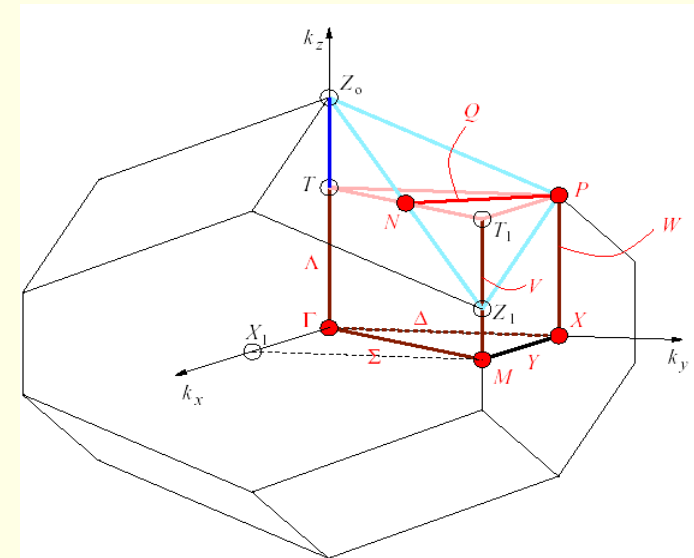


# Phonons in SBT

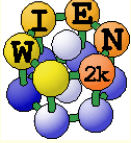


**SBT**

**CALCULATED “PHONON”  
BRANCHES**



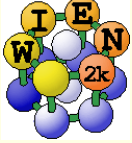




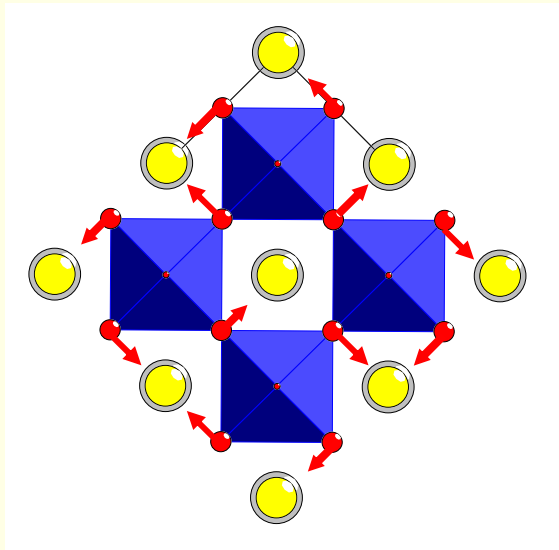
**Ferroelectric Phase = mode Eu + mode X3- + mode X2+**

**In general, good agreement with experimental frozen modes**

**Except !.... The experimental frozen mode  $X_2^+$**



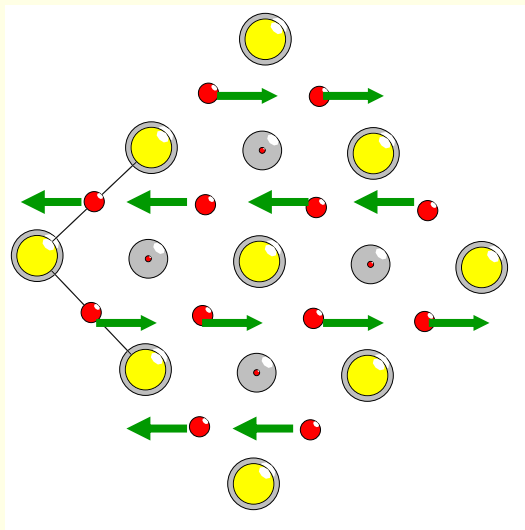
# Comparing exp. and calculated $X_2^+$ Modes



coincides approx. with  
calculated soft-mode  $X_2^+$

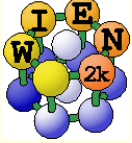
“Part 1”

Two independent normal-  
modes

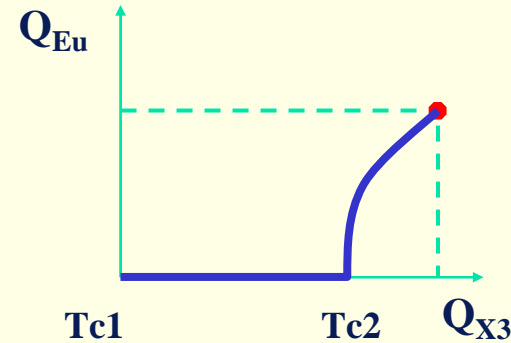
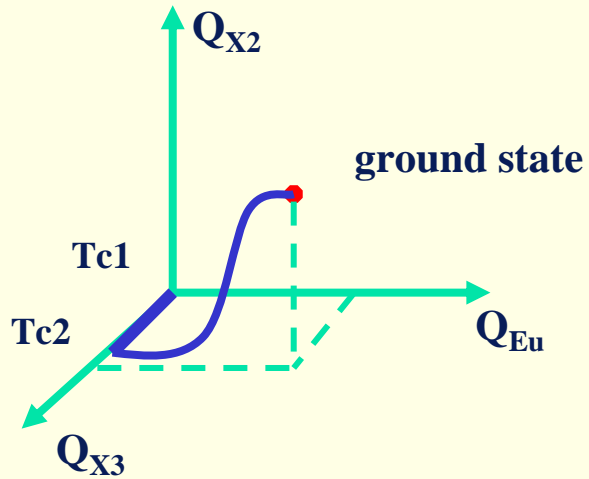


“Part 2”

coincides approx. with a  
second (hard) mode  $X_2^+$



# Energy Landscape around the tetragonal configuration

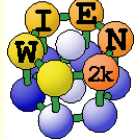


$$\text{Energy} = E_0 + \frac{1}{2} \kappa_{Eu} Q_{Eu}^2 + \frac{1}{2} \kappa_{X3} Q_{X3}^2 + \frac{1}{2} \kappa_{X2} Q_{X2}^2 + \dots$$

$$\kappa_{Eu} < 0, \quad \kappa_{X3} < 0$$

Transition Sequence:

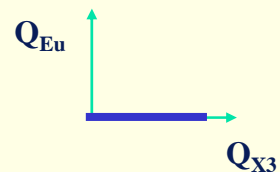
$$\text{Thermal Free Energy} = F_0 + \frac{1}{2} \alpha_{Eu}(T) Q_{Eu}^2 + \frac{1}{2} \alpha_{X3}(T) Q_{X3}^2 + \frac{1}{2} \alpha_{X2} Q_{X2}^2 + \dots$$



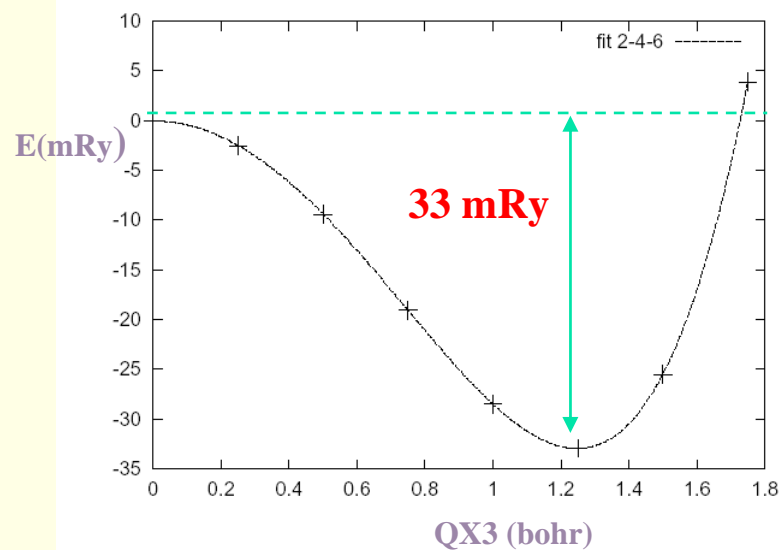
# ENERGY MAP – SBT

Main axes

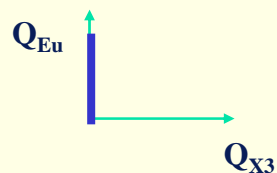
Space Group Amam



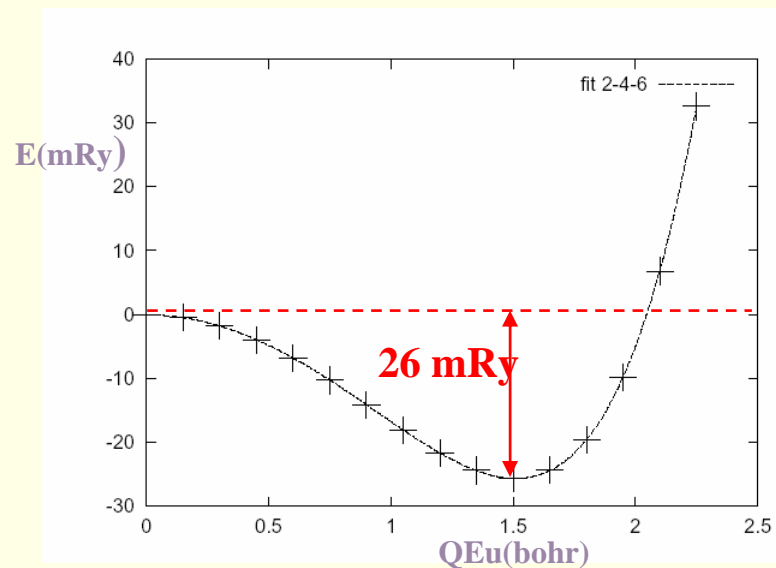
$$E_{X3} = -40.2 Q_{X3}^2 + 10.9 Q_{X3}^4 + 0.87 Q_{X3}^6$$

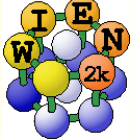


Space Group F2mm



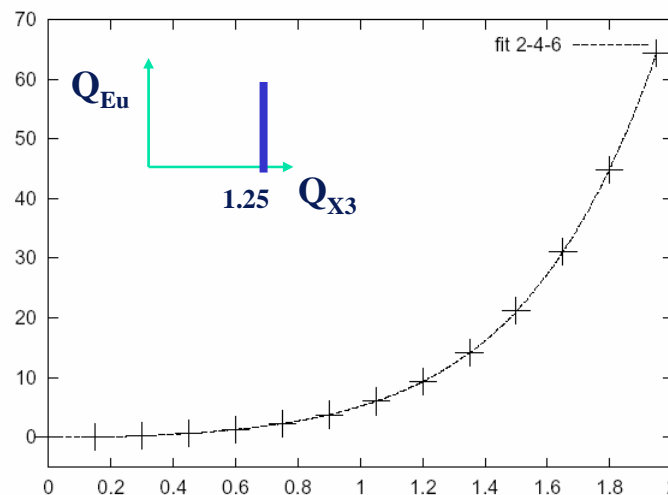
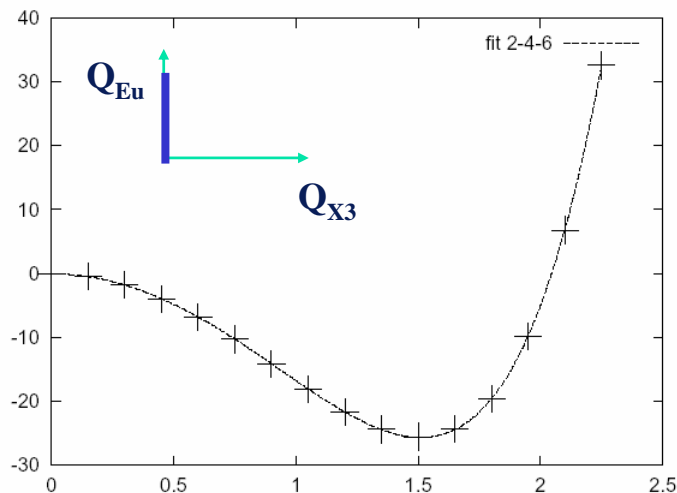
$$E_{Eu} = -19.9 Q_{Eu}^2 + 2.6 Q_{Eu}^4 + 0.52 Q_{Eu}^6$$





# ENERGY MAP – SBT:

# Coupling Eu –X<sub>3</sub><sup>-</sup>

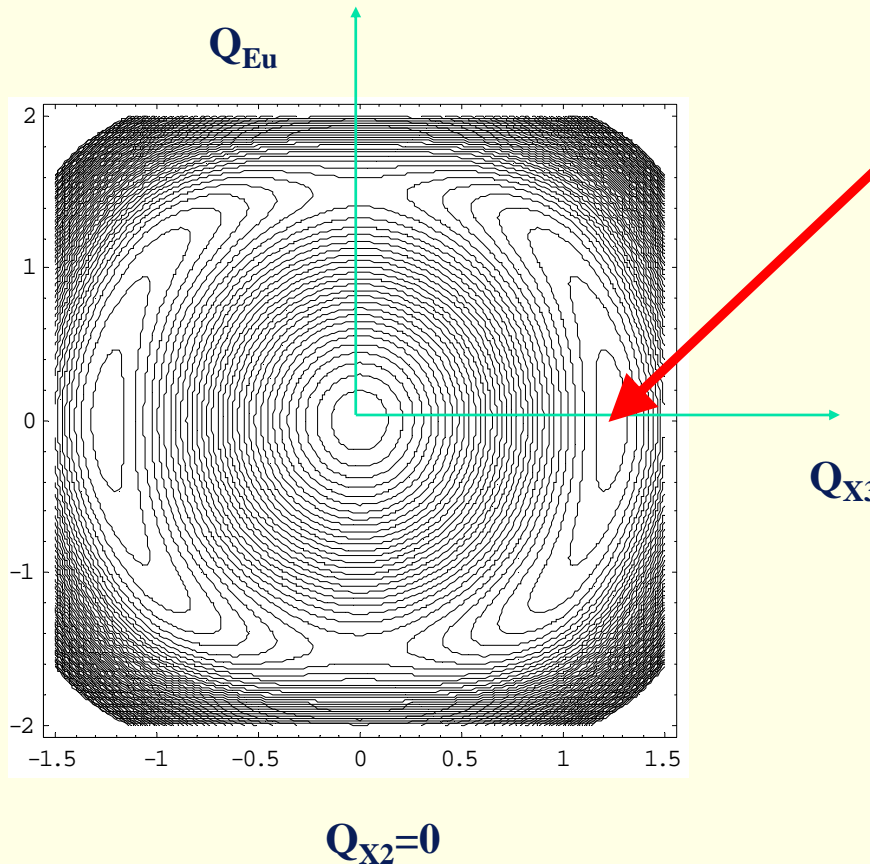
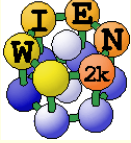


$$E_{Eu} = -19.9 Q_{Eu}^2 + 2.6 Q_{Eu}^4 + 0.51 Q_{Eu}^6$$

$$E_{Eu} = +3.1 Q_{Eu}^2 + 1.6 Q_{Eu}^4 + 0.53 Q_{Eu}^6$$

$$+14.7 Q_{Eu}^2 Q_{X3}^2 - 0.63 Q_{Eu}^4 Q_{X3}^2$$

Strong biquadratic coupling Eu –X<sub>3</sub><sup>-</sup>



**Predicted Ground State:**

**Only mode  $X_3^-$  frozen !**

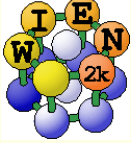
**Space group Amam**

$Q_{X3}$

**Essential reason:  
Strong biquadratic coupling  
penalizes “mixed” states**

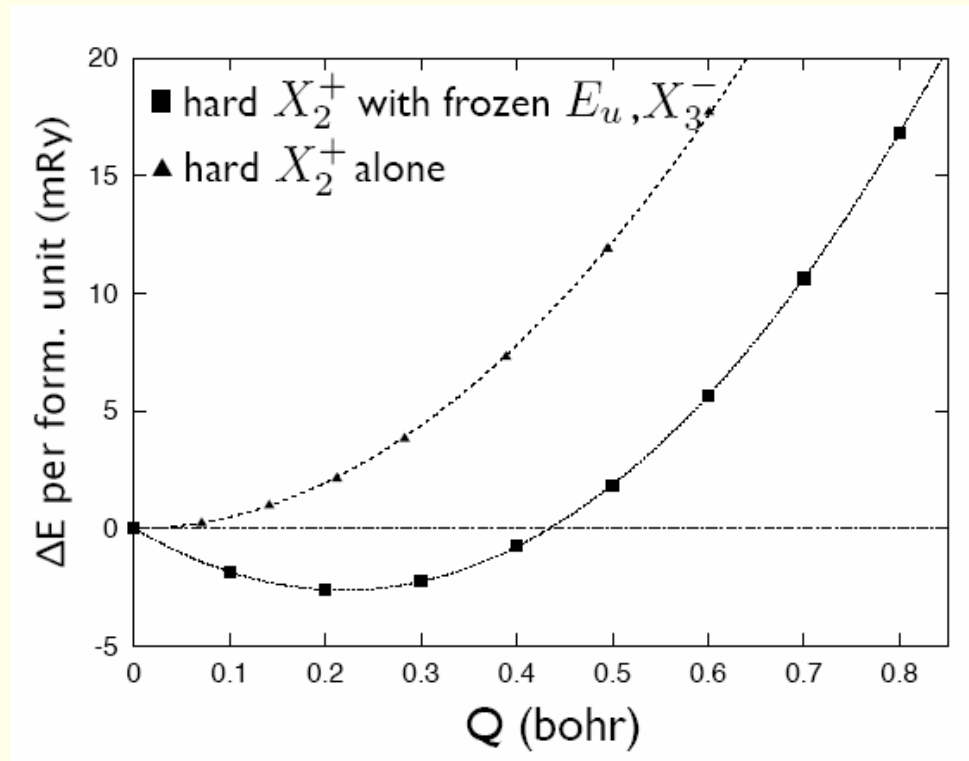
**Something missing?**

**•coupling with  $X_2^+$**

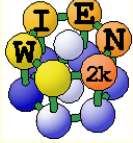


# Coupling with $X_2^+$

- Coupling with **soft**  $X_2^+$  mode relatively weak !
- Strong renormalization of **hard (second)**  $X_2^+$  mode



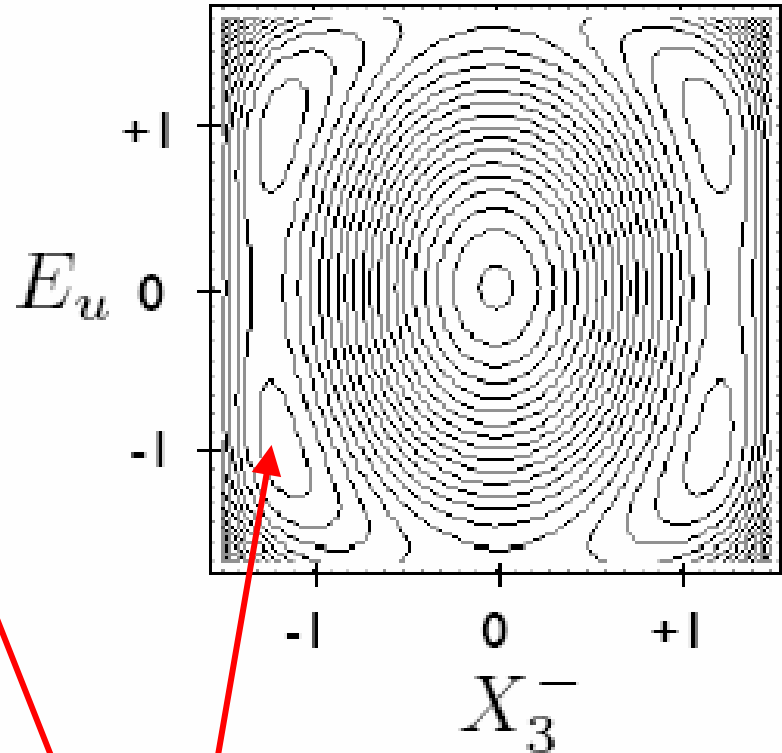
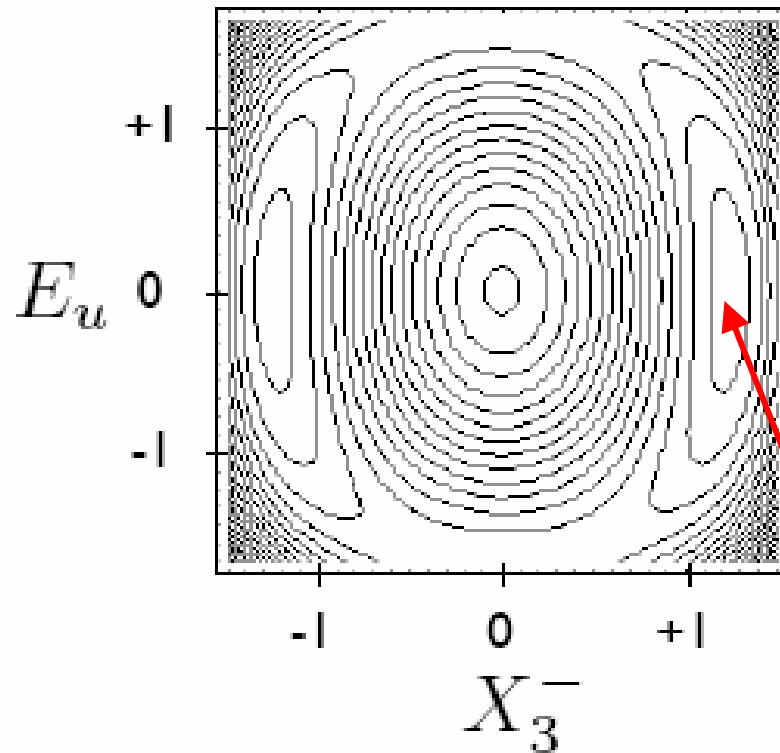




# Energy landscape in SBT

including only **soft**  $X_2^+$  mode

with additional **hard**  $X_2^+$  mode

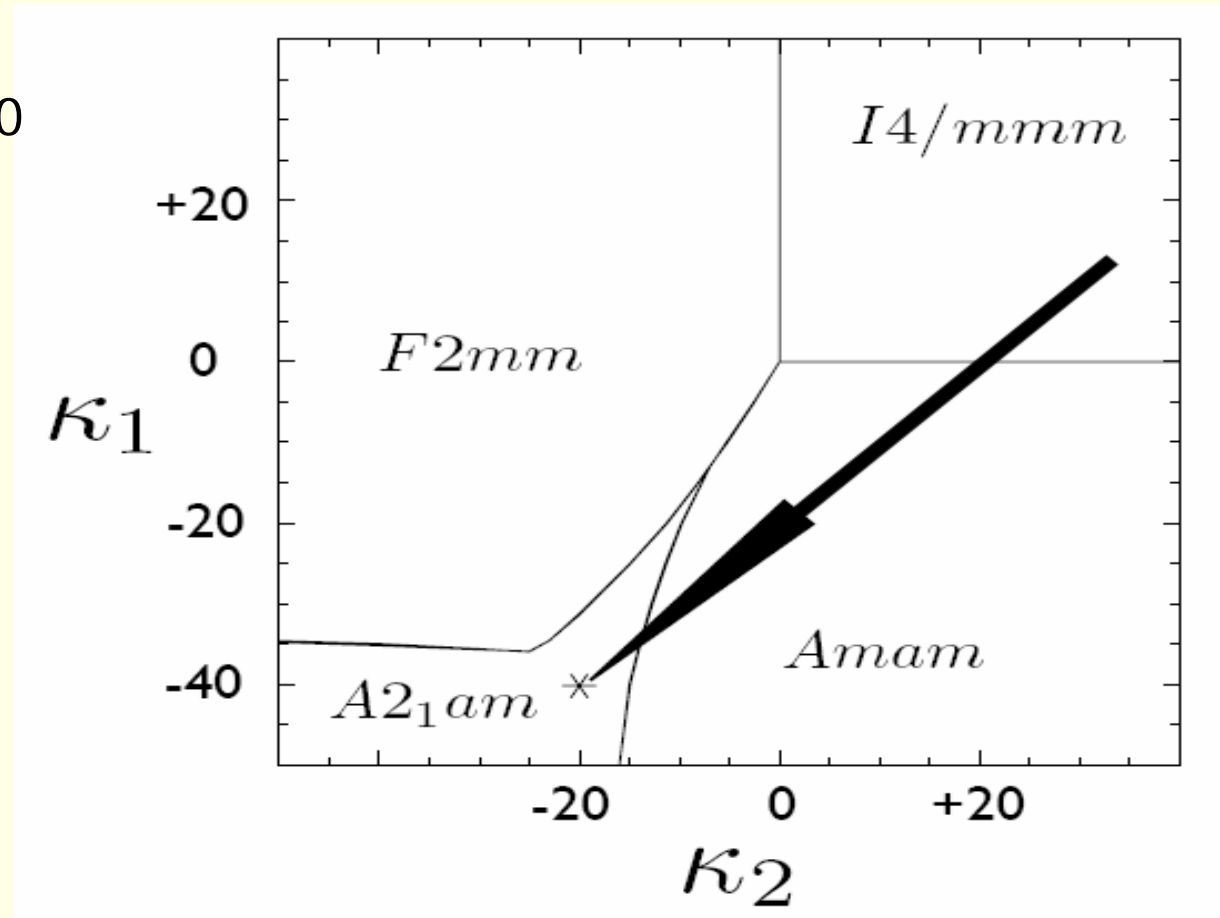


only with the **additional hard**  $X_2^+$  mode the experimentally observed combination of **3 different modes** can be established

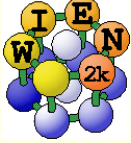


# Phase diagram for $\text{Bi}_2\text{SrTa}_2\text{O}_9$ - SBT

- Finite temperature renormalization of the  $T=0$  energy map
- Landau theory suggests linear  $T$ -variation of the quadratic stiffness coefficients  $\kappa_{\text{Eu}}$  and  $\kappa_{\text{X}3^-}$

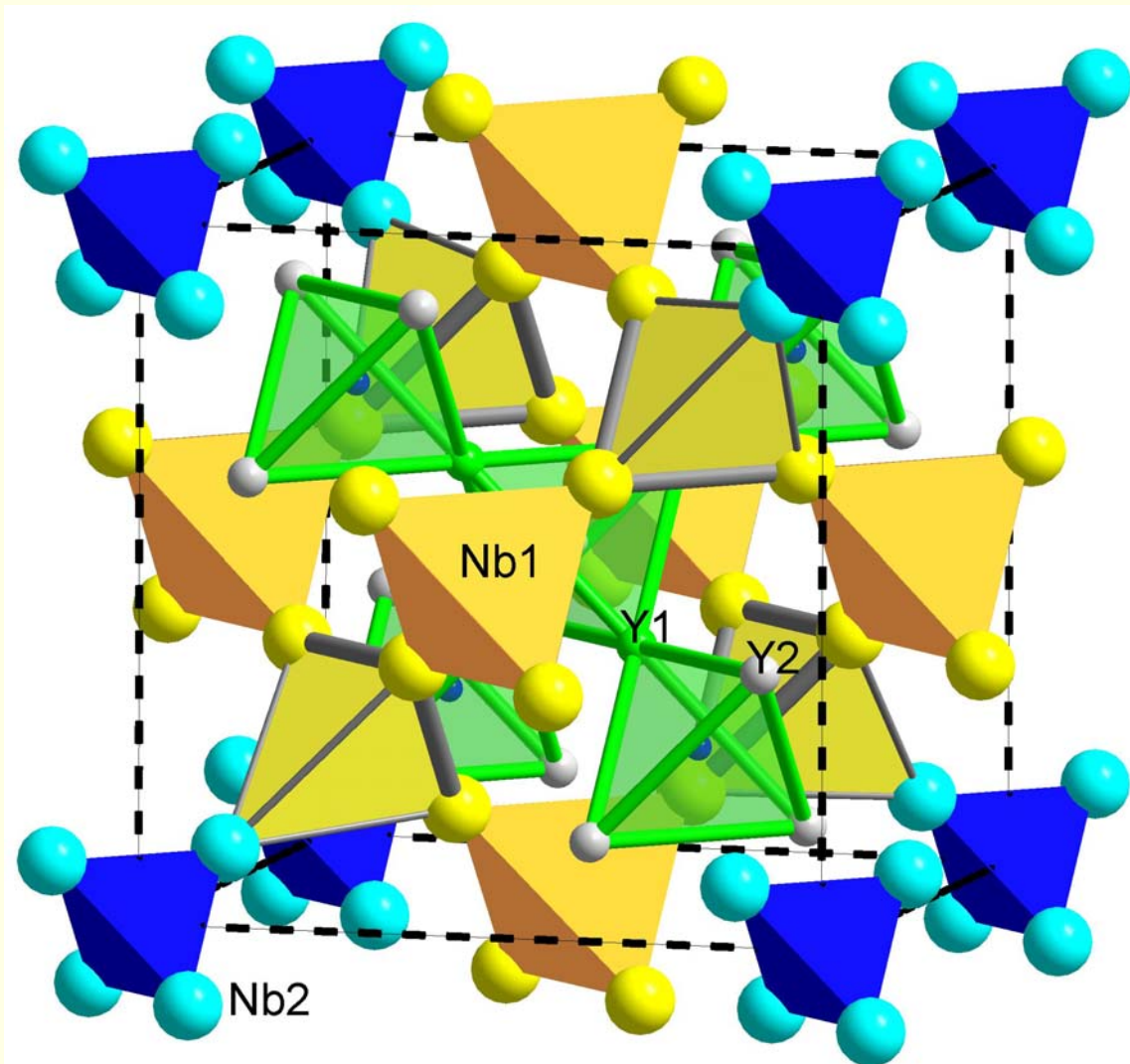


The topology of this phase diagram predicts **two second order** phase transitions and implies the existence of an **intermediate phase** of  $Amam$  symmetry. A **single first order** PT is impossible.



# Pyrochlore $\text{Y}_2\text{Nb}_2\text{O}_7$

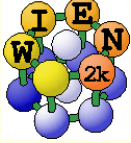
Insulating and non-magnetic **4d** TM-oxide



P. Blaha, D. Singh, K. Schwarz,  
PRL in print (2004)



Metal Sublattice:  
Corner-shared  
tetrahedral network



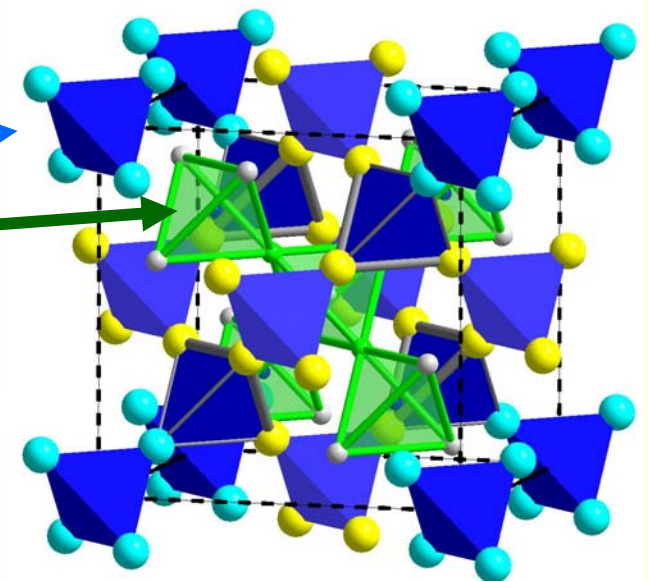
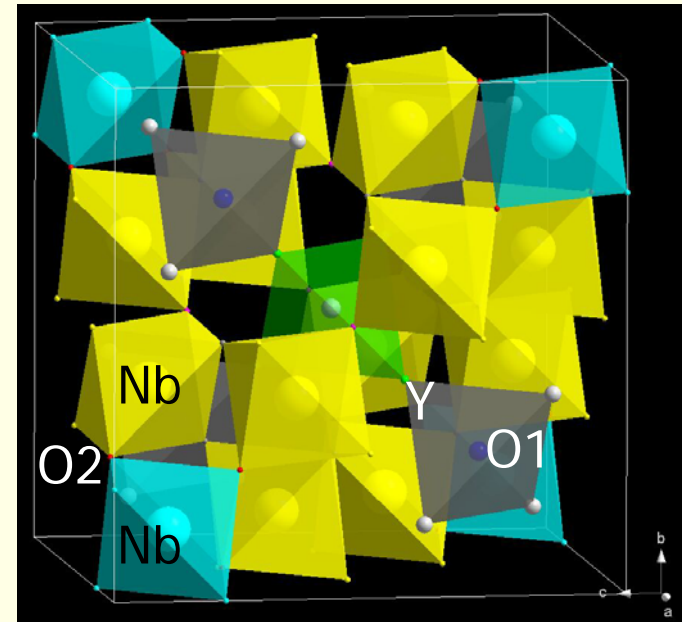
# Pyrochlore structure of $\text{Y}_2\text{Nb}_2\text{O}_7$ :

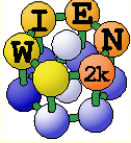
- Experimental structure from powder diffraction (Fukazawa, Maeno):

Y. Maeno et al, Nature 372, 532 (1994)

H. Fukazawa, Y. Maeno, Phys. Rev. B 67, 054410 (2003)

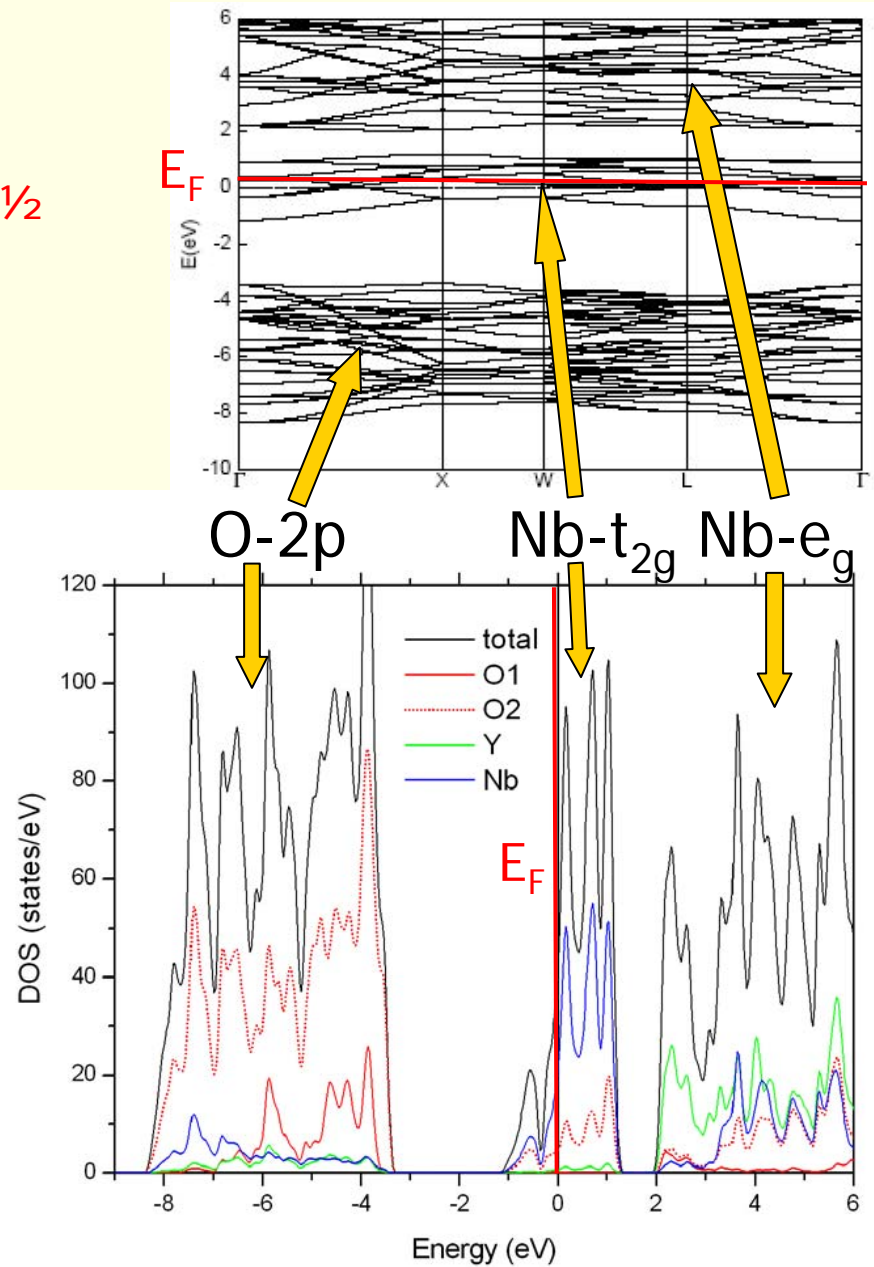
- Pyrochlore structure:  $Fd-3m$ ,  $a_0 = 10.33 \text{ \AA}$
- 22 atoms (2 FU/cell)
- One free parameter for O2:  $(0.344, 1/8, 1/8)$
- Corner shared Nb - O octahedra
- Corner shared O1 - Y tetrahedra
- Y surrounded by 8 oxygens (~ cube)
- Metal sublattices
  - Nb-tetrahedra (empty)
  - Y-tetrahedra (around O)
- Insulator
- Nonmagnetic

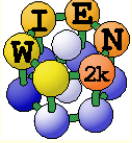




# First theoretical results:

- Ionic model:  $\text{Y}_2^{3+}\text{Nb}_2^{4+}\text{O}_7^{2-}$      $\text{Nb}^{4+}$ :  $4d^1$
  - $\rightarrow$  metallic or localized system with spin  $\frac{1}{2}$  (neither one observed in exp.)
  - LDA gives nonmagnetic metallic ground-state with conventional  $t_{2g}$ - $e_g$  splitting due to the octahedral crystal field of the oxygen atoms.
- “degenerate”  $t_{2g}$  states are only partly filled.

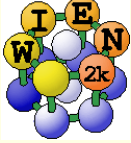




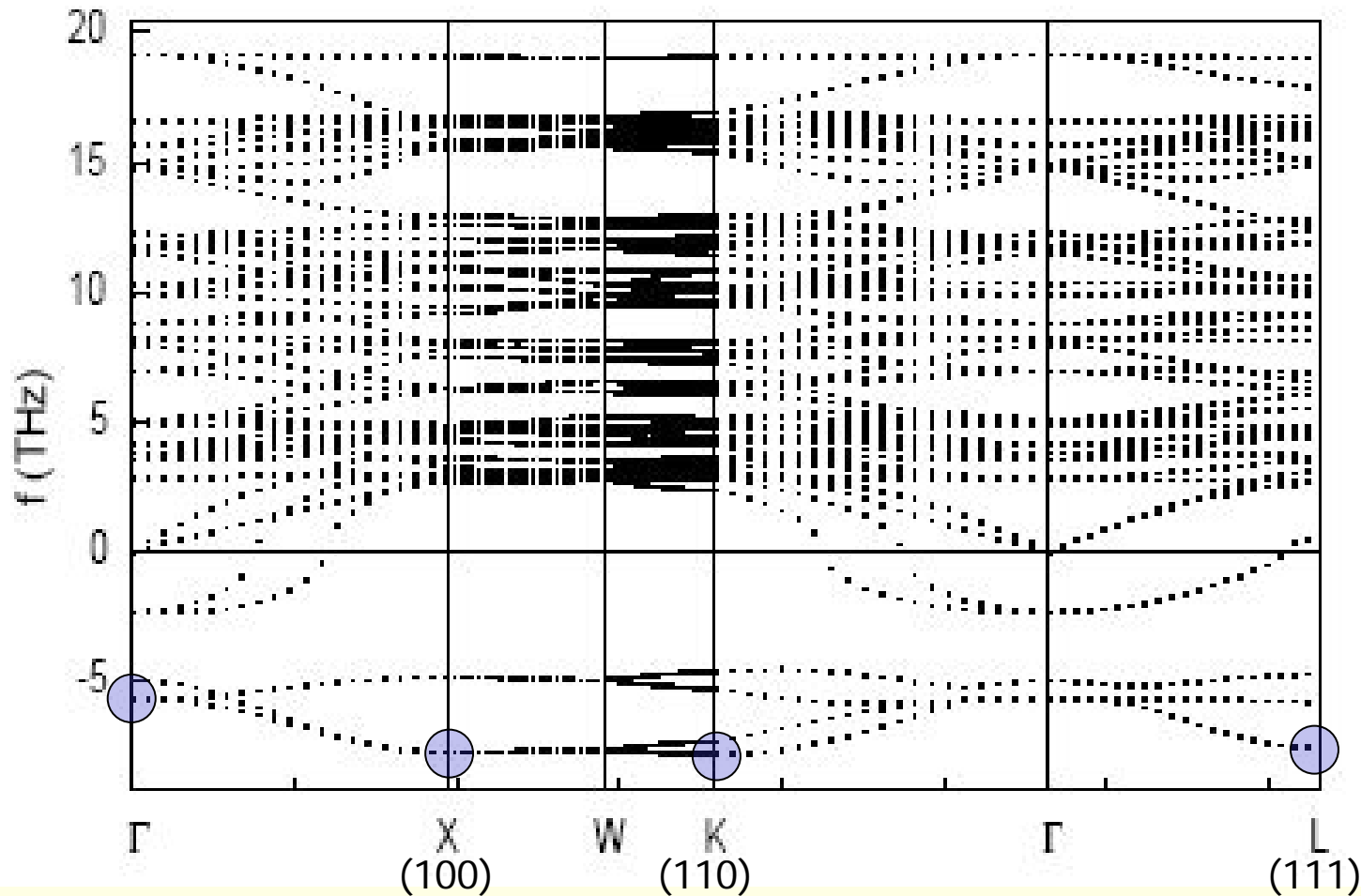
# How could one obtain a non-magnetic insulator ?

- **Antiferromagnetic  $s=1/2$  solution**
  - *(on geometrically frustrated lattice !?)*
- **Localization, strong  $e^-e^-$  correlation:**
  - ***4d (not 3d !)*** electrons,
  - *thus correlation should be small (Hubbard- $U \sim 2-3$  eV)*
  - *LDA+ $U$  with  $U=6$  eV gives insulator (FM ground state, no AFM)*
  - *bandwidth of  $t_{2g}$  bands: 2.5 eV (similar to  $U$ )*
- **structural distortion, which breaks the dominant octahedral crystal field**
- ➔ **Search for phonon-instabilities**
  - *88 atom supercell, 46 symmetry adapted selected distortions from PHONON; resulting forces ➔ back into PHONON*



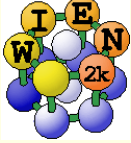


# Phonon-bandstructure of $\text{Y}_2\text{Nb}_2\text{O}_7$



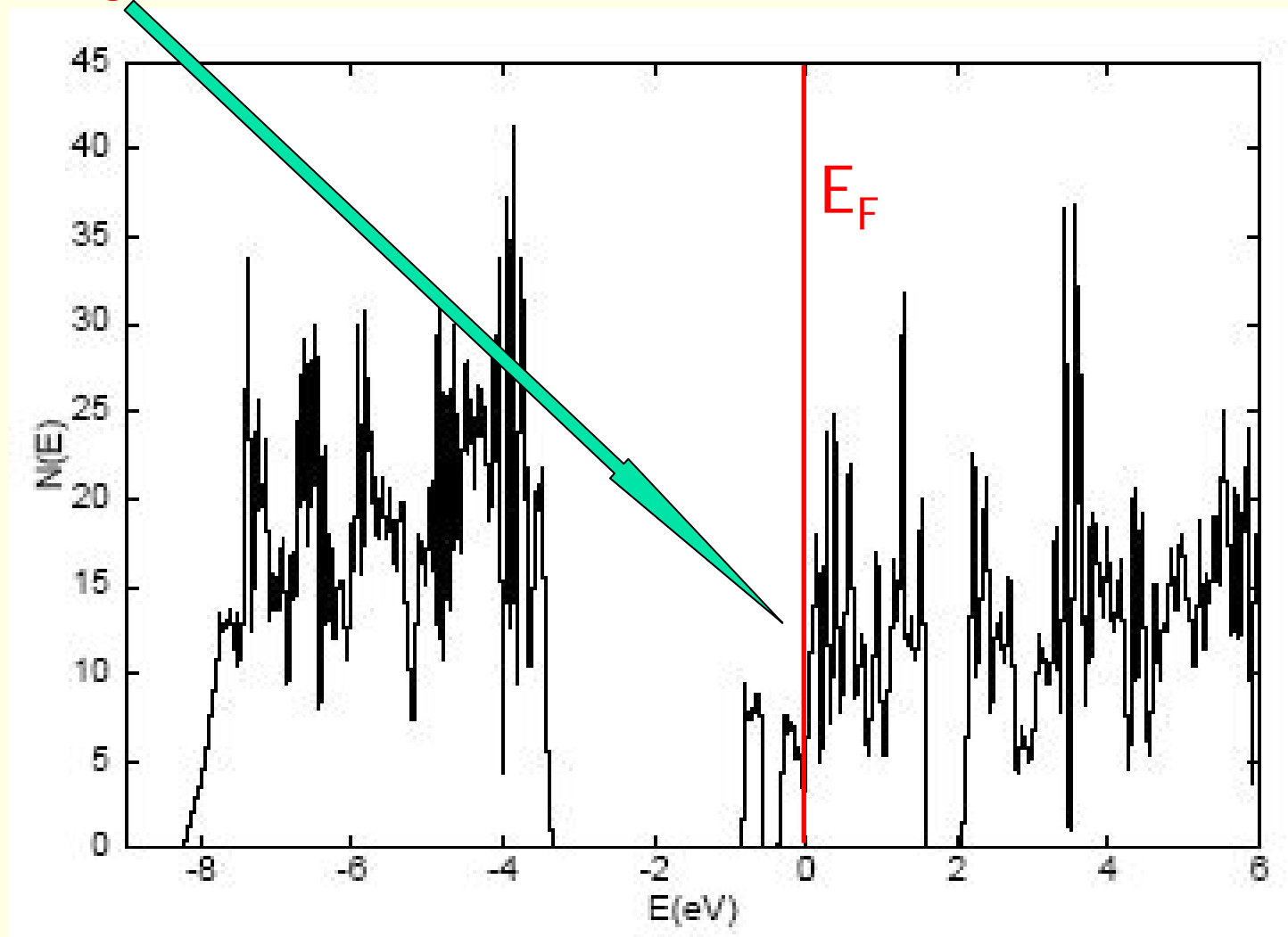
- strong Phonon-instabilities, lowest at X, K, L
- select a certain (unstable) phonon, freeze it with certain amplitude into the structure and perform full structural optimization

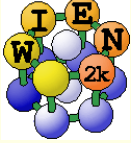




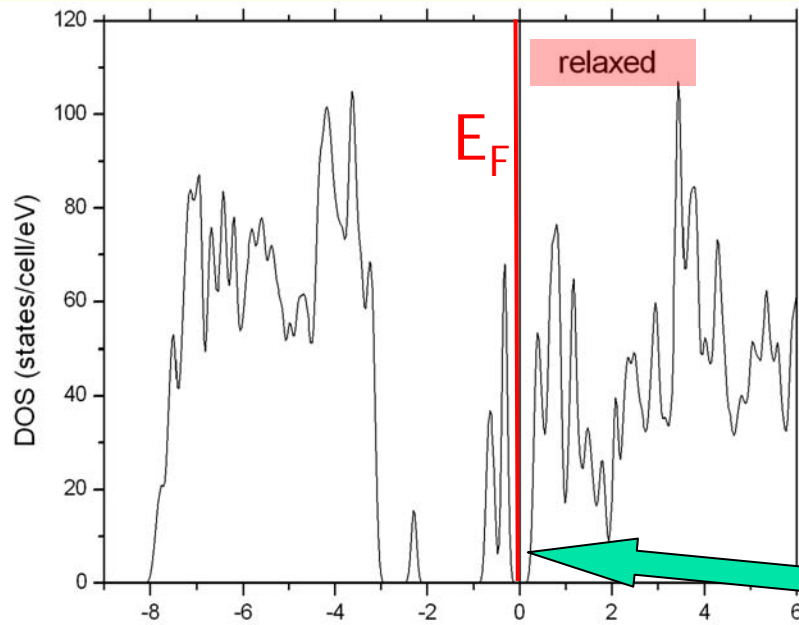
# $\Gamma$ , X and K-point phonons:

- energy lower than in ideal pyrochlore structure, but still not insulating

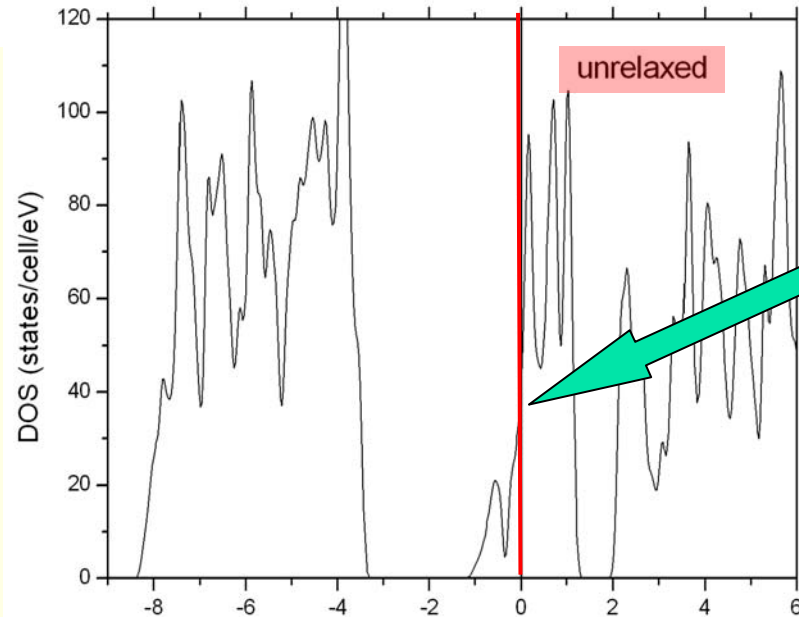




# L-point (111) phonon:

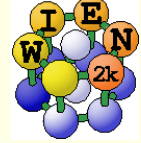


- Relaxed structure is an **Insulator**
- energy gain of 0.5 eV/FU



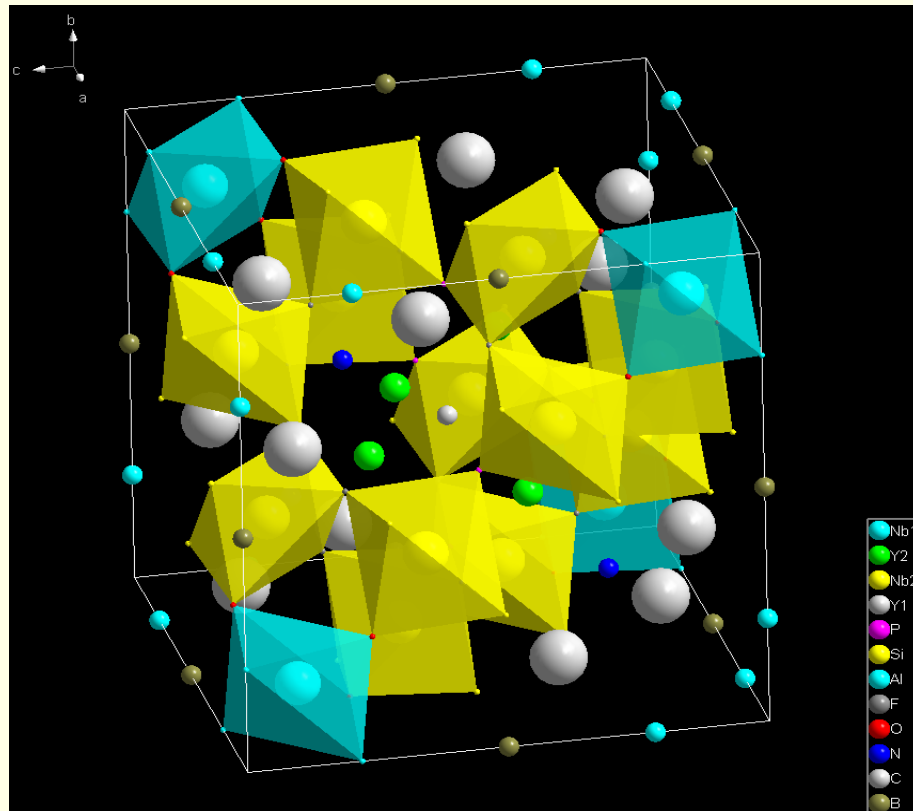
gap

metal

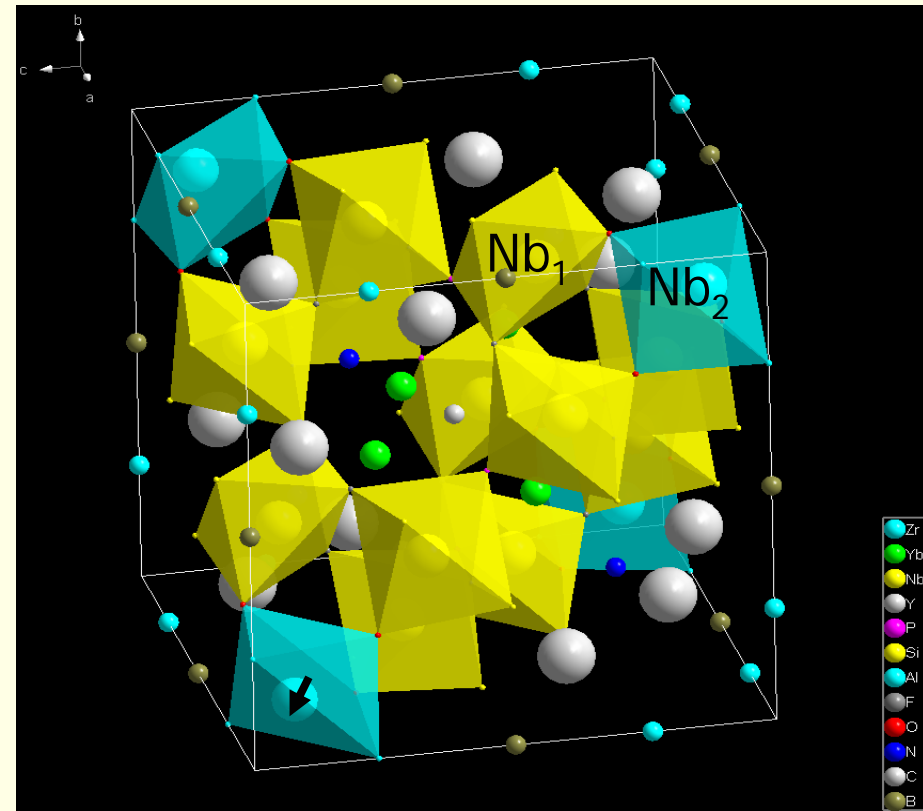


# Relaxed structure:

- Primitive supercell with 88 atoms
- all atoms inequivalent
  - due to numerical optimization of the positions in P1
- **Symmetrization using KPLOT** (R.Hundt, J.C.Schön, A.Hannemann, M.Jansen: *Determination of symmetries and idealized cell parameters for simulated structures, J.Appl.Cryst. 32, 413-416 (1999)*)
  - *Tests possible symmetries with increasing tolerance*
- Space group → P-43m, 88 atoms/cell,
  - *Inequivalent atoms:*
    - 2 Y
    - 2 Nb
    - 3 O1
    - 5 O2



Experimental (ideal)

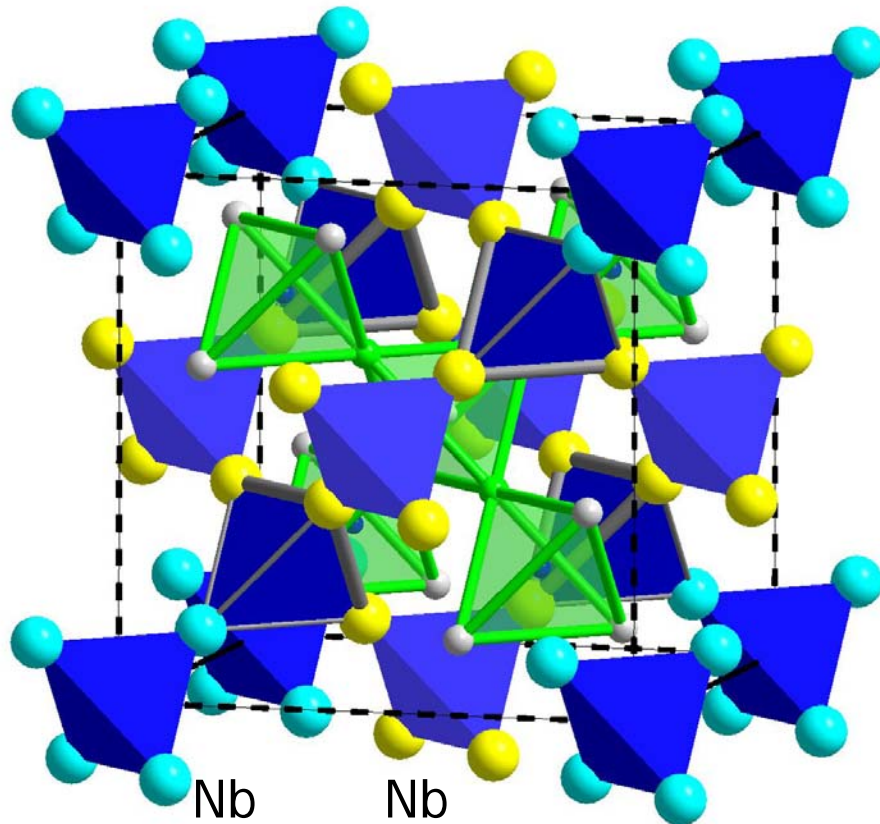


relaxed:

Nb<sub>2</sub> moves „off-center“, O octahedra changes little, Nb<sub>2</sub>-Nb<sub>2</sub> bonds ?

# Main change in structural relaxation

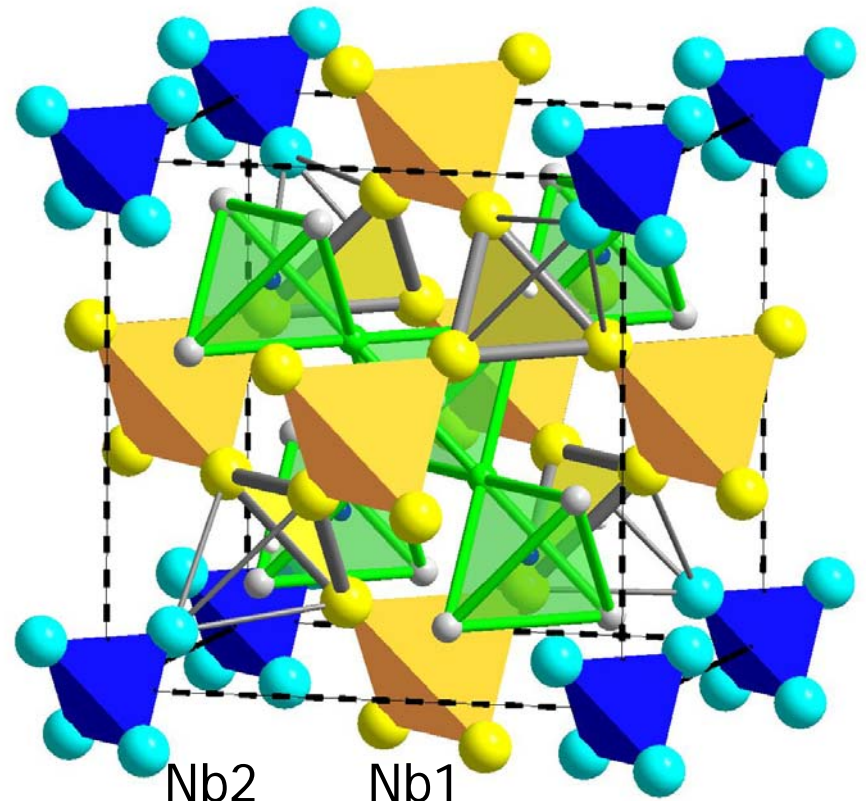
Original pyrochlore



3.65 Å

equal

Relaxed structure



small

2.91 Å

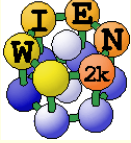
large

3.90 Å

Nb1 - Nb2

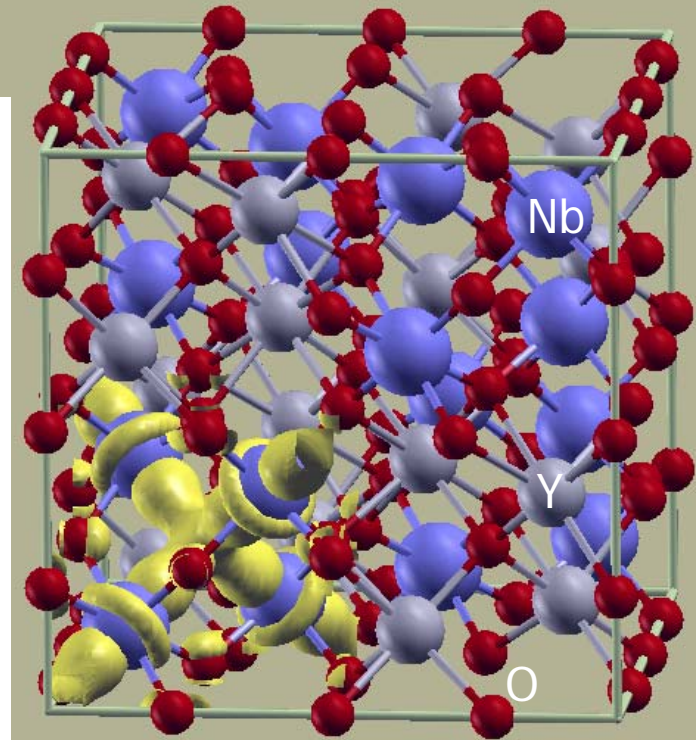
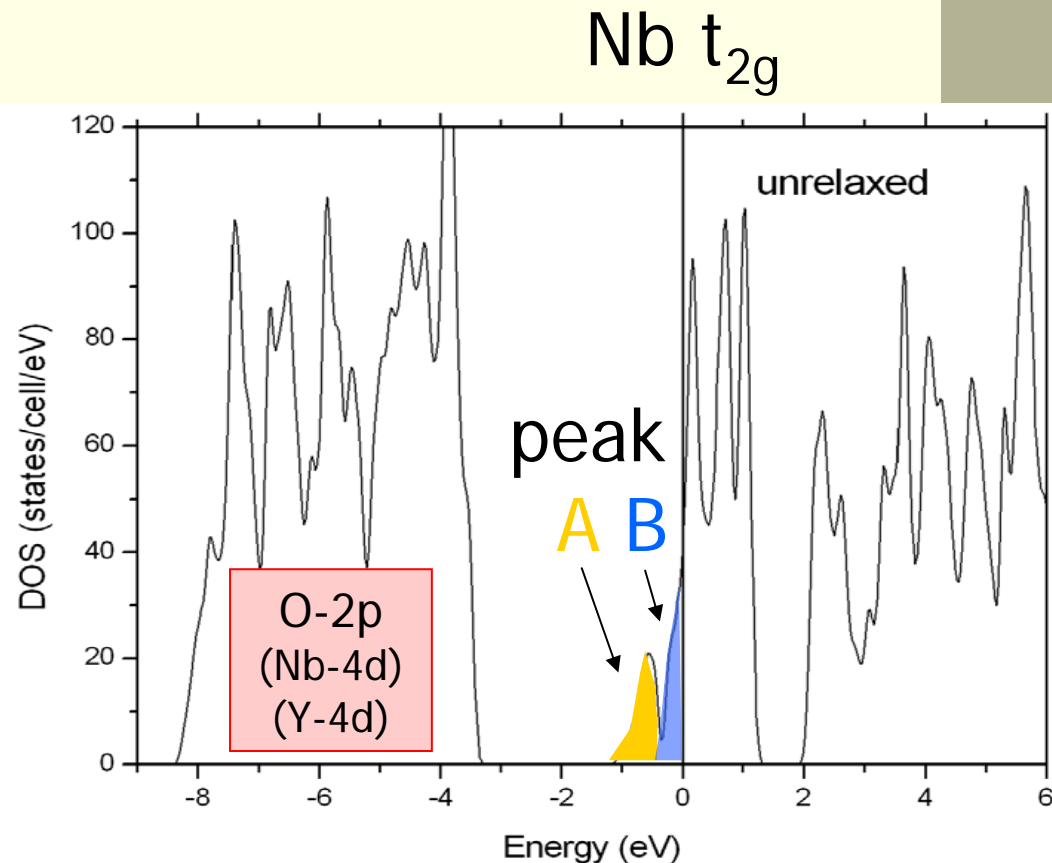
3.89 Å

Nb1-triangle: 3.40 Å

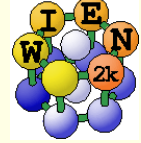


# Chemical bonding: original structure

peak A: Nb-Nb bonds within the Nb tetrahedra by  $d-z^2$  orbitals

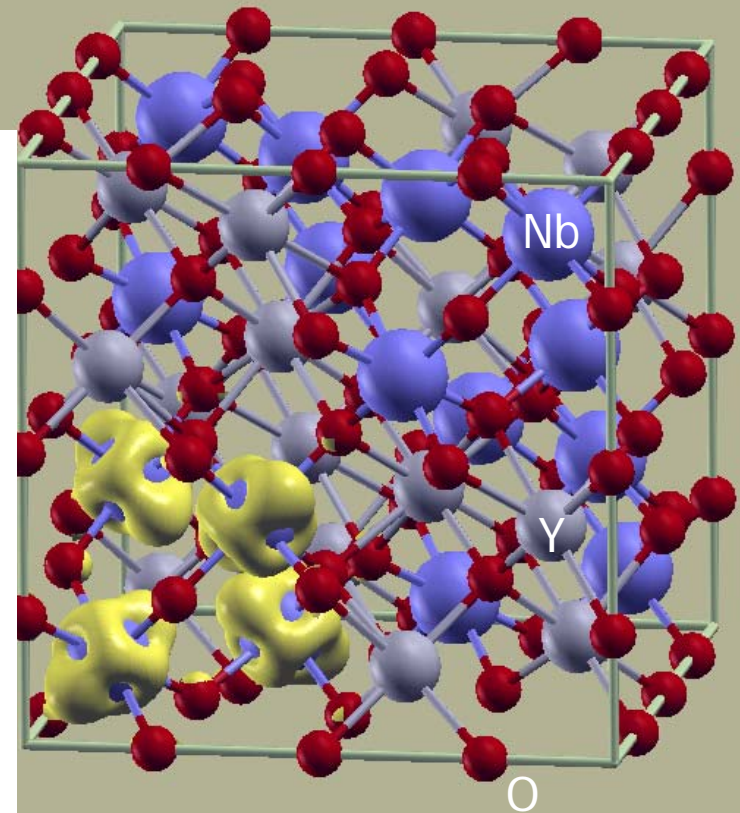
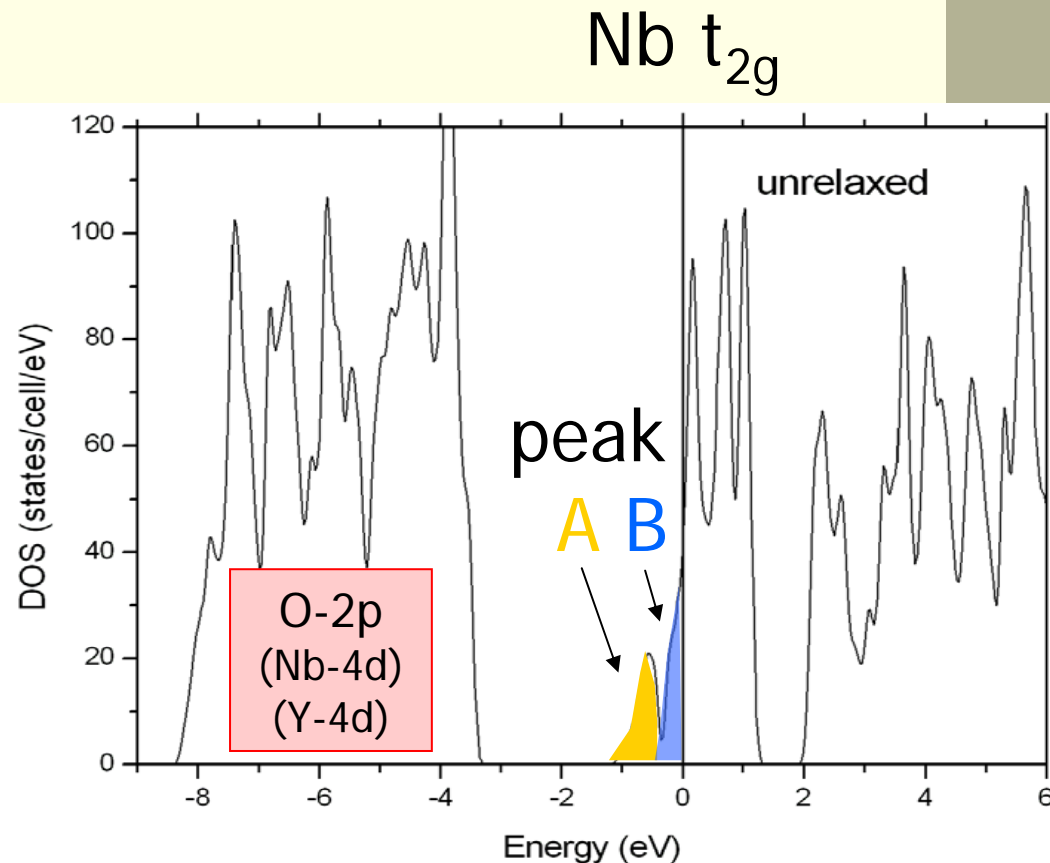






# Chemical bonding: original structure

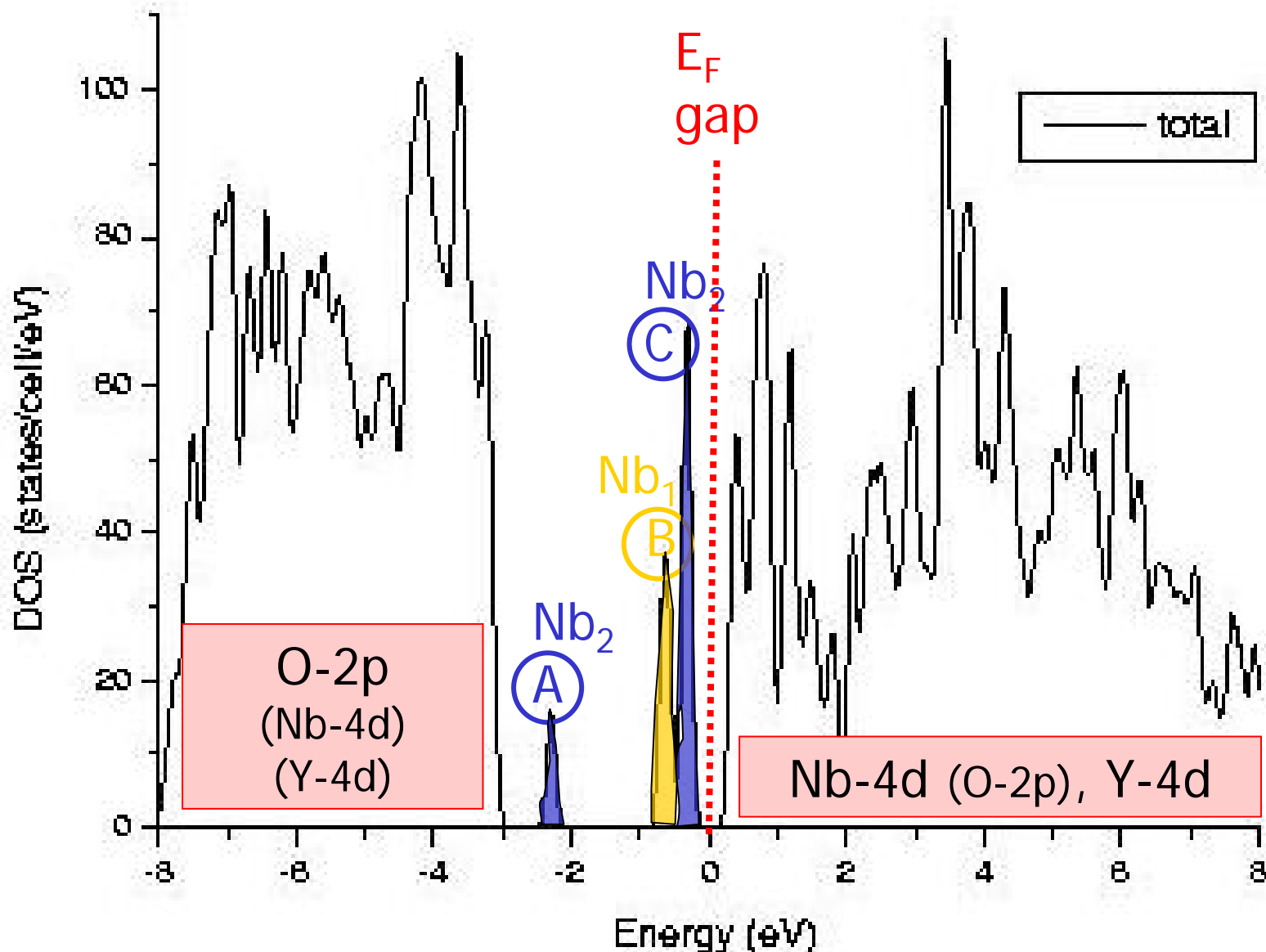
peak B: mixture of all 3  $t_{2g}$  orbitals

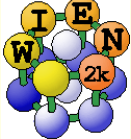




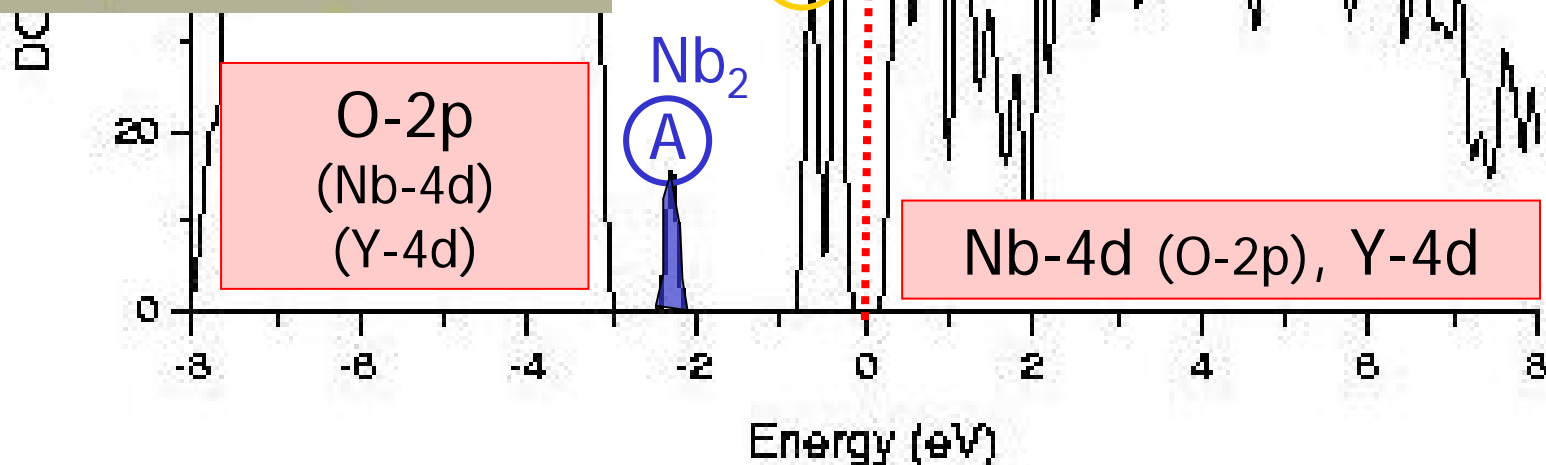
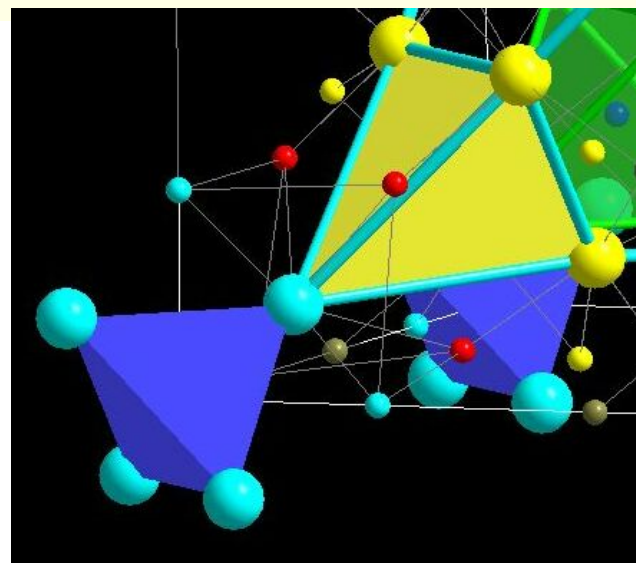
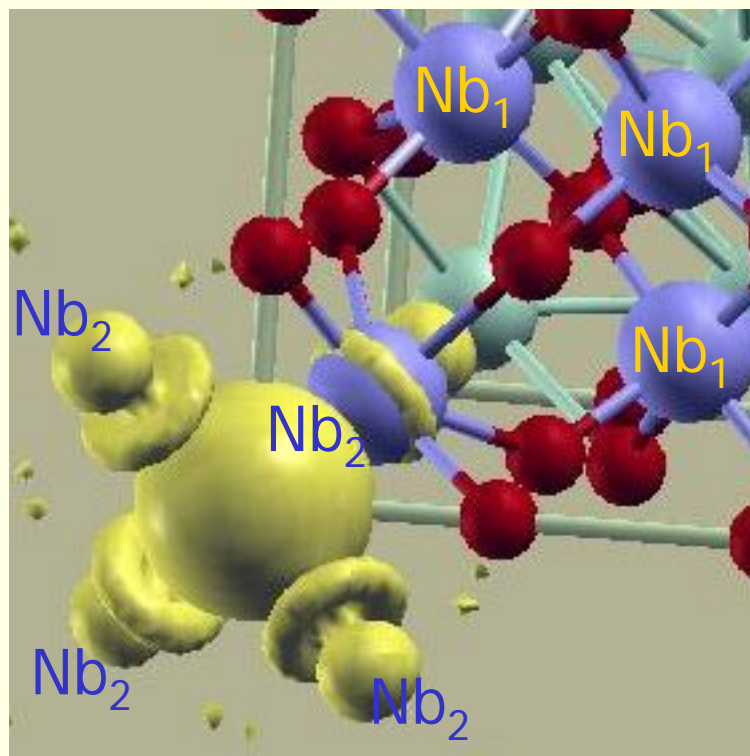


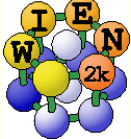
# DOS of relaxed structure



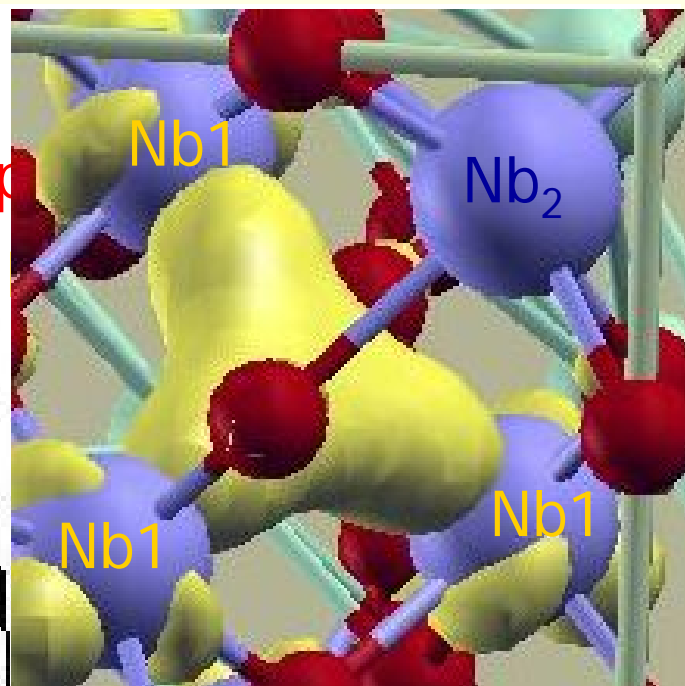
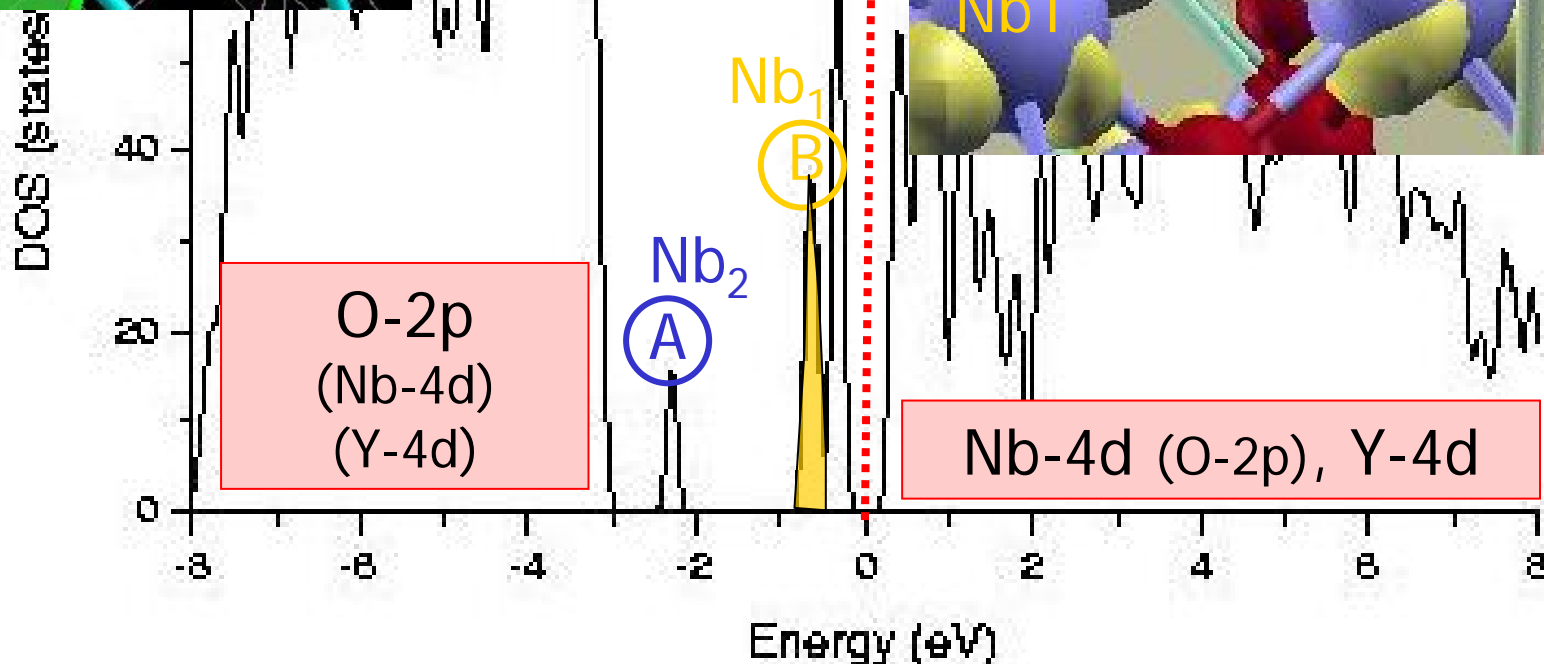
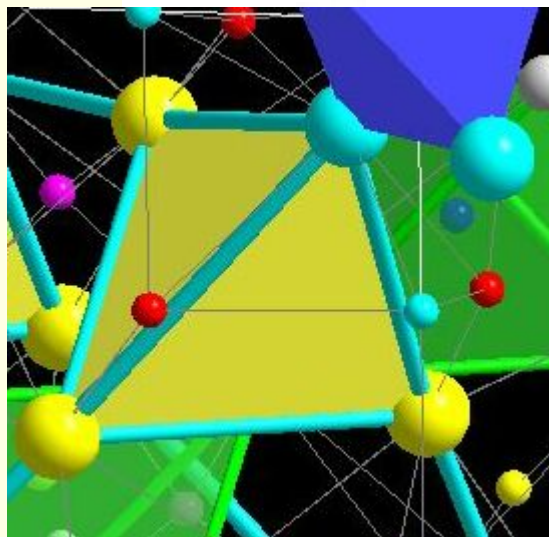


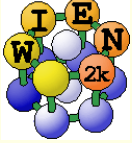
# Peak A ( $\text{Nb}_2$ ) 4-center bond ( $d\text{-}z^2$ )



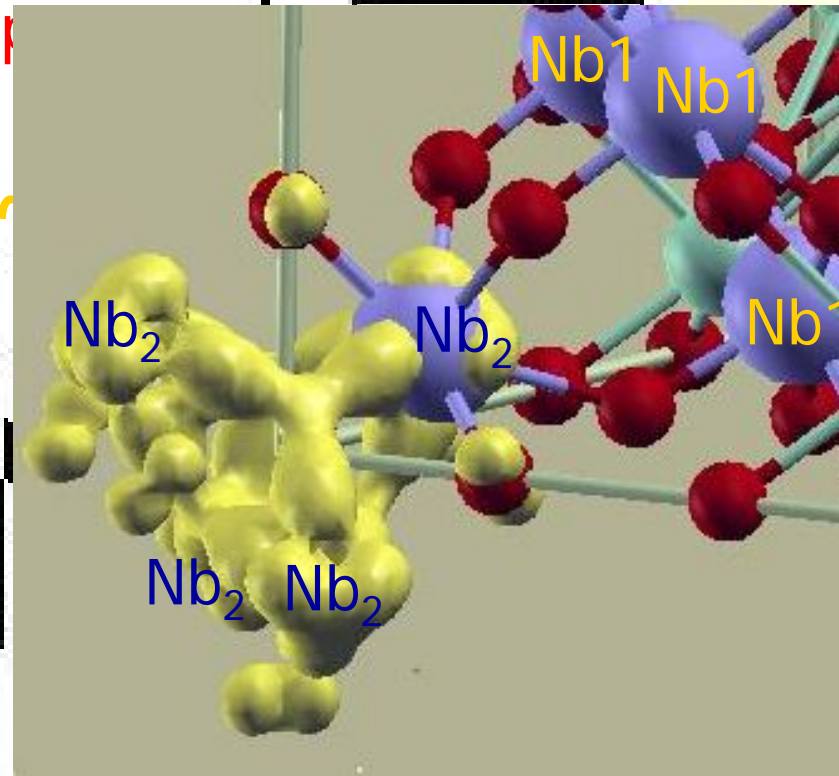
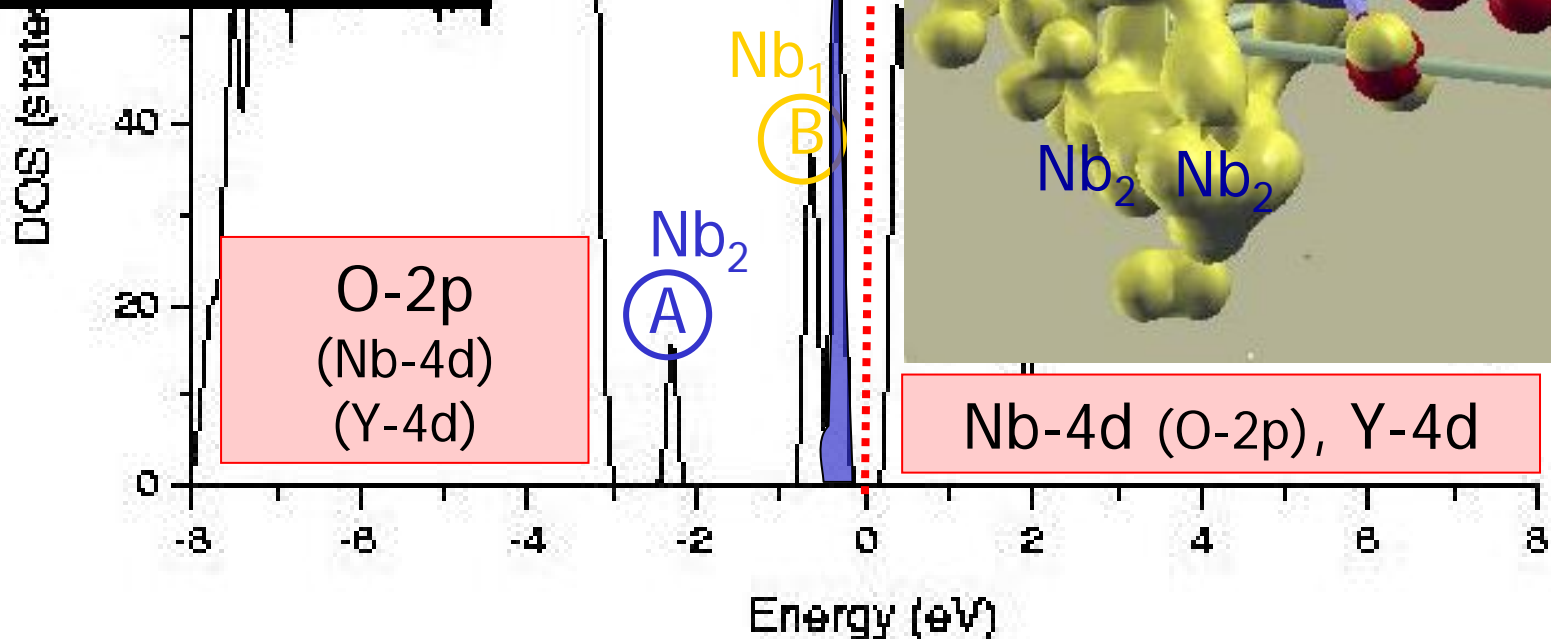
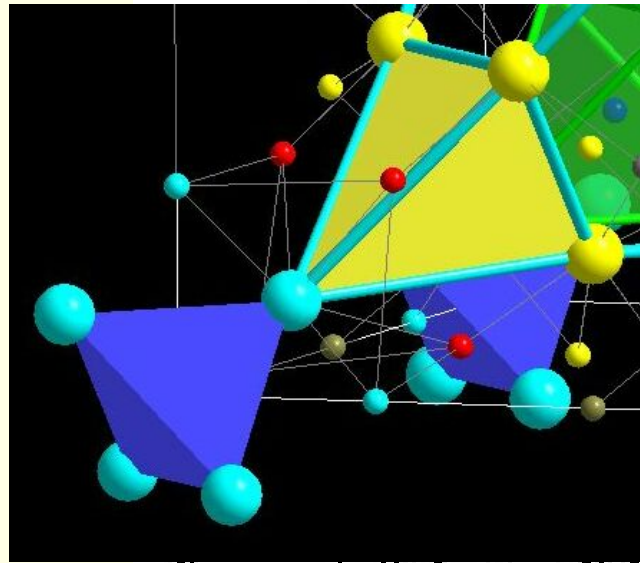


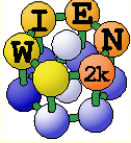
# Peak B ( $\text{Nb}_1$ ) 3-center bond (new !)





# Peak C ( $\text{Nb}_2$ ) 3-center bonds on all 4 faces

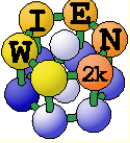




# Results for $Y_2Nb_2O_7$

- Orbital ordering of Nb-4d orbitals and structural distortion !
  - *Can be described with LDA !!*
- M-M interaction (second NN): 2 types of bonds:
  - *strong Nb-Nb 4-center bonds (only Nb2  $d_{z^2}$ )*
  - *strong Nb-Nb 3-center bonds (both, on Nb1 and Nb2)*
- Relaxation
  - *Two types of Nb*
    - Nb1 (12i) highly elongated tetrahedron along (111)
    - Nb2 (4e) perfect tetrahedron with short distances
- Metal/insulator
  - *Ideal structure metallic, relaxed structure insulating*

P.Blaha, D.J.Singh, K.Schwarz,  
*Charge singlets in pyrochlore  $Y_2Nb_2O_7$*   
Phys.Rev.Lett. (in print) (2004)



# Advantage/disadvantage of WIEN2k



- + robust all-electron full-potential method
- + unbiased basisset, one convergence parameter (LDA-limit)
- + all elements of periodic table (equal expensive), metals
- + LDA, GGA, meta-GGA, LDA+U, spin-orbit
- + many properties
- + w2web (for novice users)
- ? speed (+ memory requirements)
  - + *very efficient basis for large spheres (2 bohr) (Fe: 12Ry, O: 9Ry)*
  - *less efficient for small spheres (1 bohr) (O: 25 Ry)*
  - *large cells ( $n^3$ , iterative diagonalization not perfect)*
- no stress tensor
- no linear response

Thank you for  
your attention !