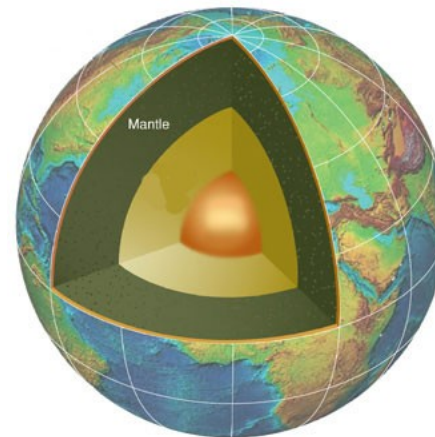




# Deep Earth thermodynamics from phonons

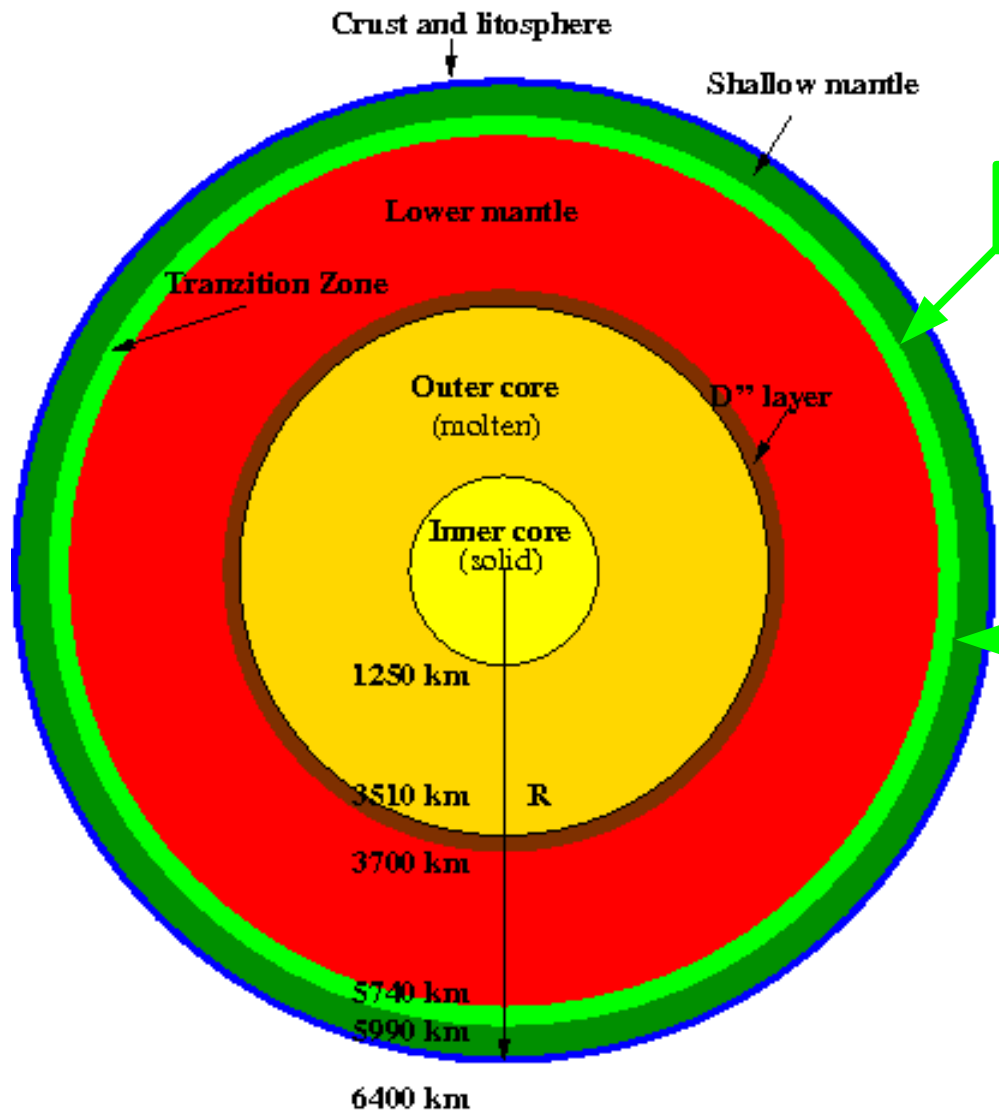


Kamil Tokár

Department of Material Research by Computers

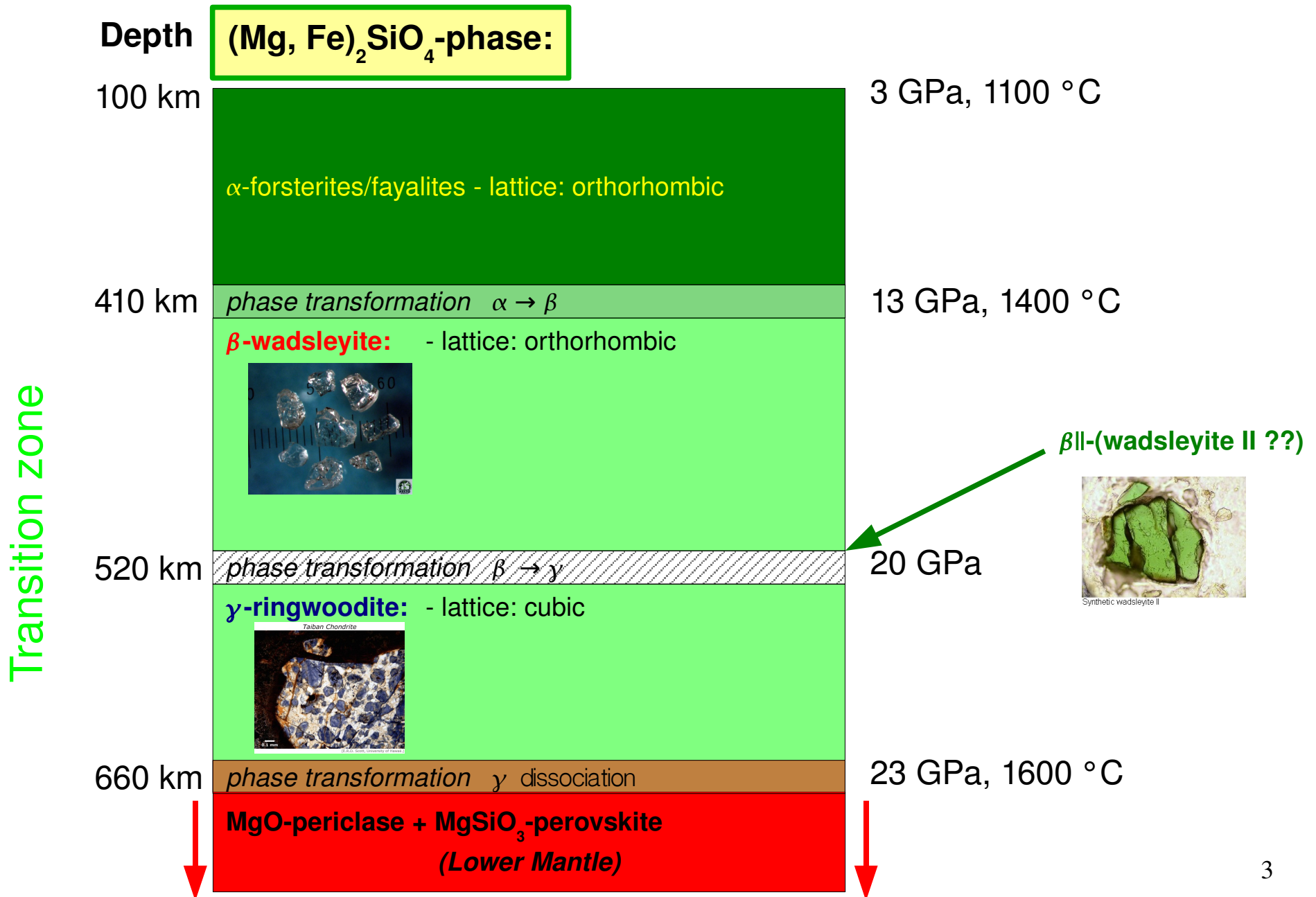
**Kraków, Poland 1-4<sup>th</sup> DEC 2010**

# Structure of Earth interior - overview



	<b>depth</b>
Continental crust:	0-75 km
Oceanic crust:	0-10 km
Upper mantle:	10-400 km
Transition zone:	400-650 km
Lower mantle:	650-2 890 km
D" layer:	2 700-2 890 km
Outer core:	2 890-5 150 km
Inner core:	5 150-6 370 km

# Structure of Earth's Transition Zone – simplified description



## Wadsleyite against Wadsleyite II

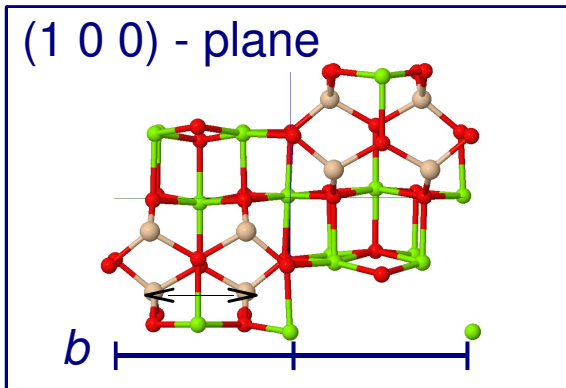
Wadsleyite II –  $\text{Mg}_2\text{SiO}_4$  is a recently detected magnesium-iron silicate phase - a potential host for hydrogen in the Transition Zone, between the wadsleyite and ringwoodite zones (Smyth J.R. and Kawamoto T. (1997)).

Differences

Space symmetry:  $Imma$

$a=5.698$   $b=11.462$   $c=8.257$  [Å]

(1 0 0) - plane

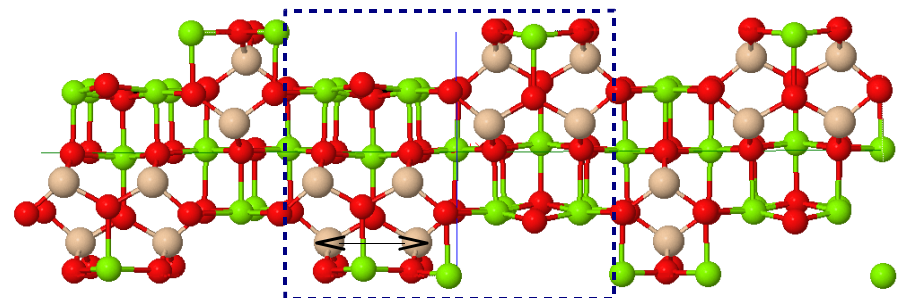


Wadsleyite

$a=5.749$   $b=28.791$   $c=8.289$  [Å]

Wadsleyite II  
(1 0 0) - plane

● Oxygen  
● Magnesium  
● Silicon



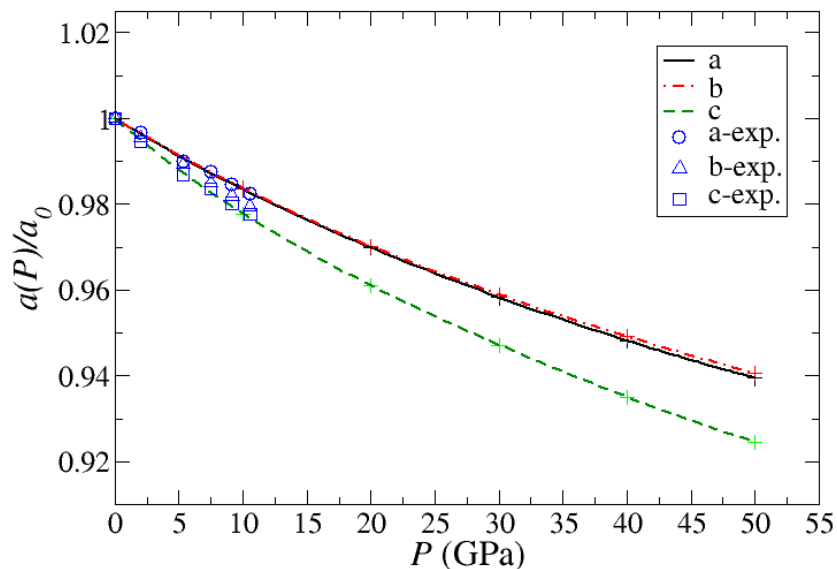
$b$

## Methods used in theoretical study of Wadsleyite II

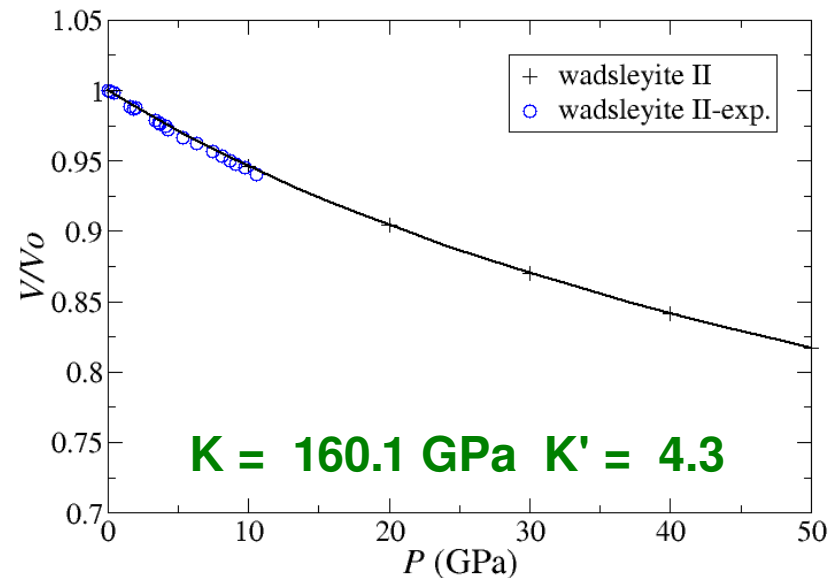
- DFT+GGA with PBE functional and PAW approach
- Direct method for determination of the lattice dynamics
- Quasi-harmonic Approximation (QHA) in Thermodynamic model

# Unit cell parameters of Wadsleyite II – pressure dependence

Lattice parameters



3<sup>rd</sup> – order Birch-Murnaghan EOS



Comparison of lattice constants

Wadsl. II	$a$ [Å]	$b$ [Å]	$c$ [Å]
10.56 GPa <sup>a)</sup>	<b>5.583</b>	<b>28.426</b>	<b>8.056</b>
10.00 GPa <sup>b)</sup>	<b>5.654</b>	<b>28.322</b>	<b>8.105</b>
0.00 GPa <sup>b)</sup>	<b>5.749</b>	<b>28.791</b>	<b>8.289</b>

Comparison of bulk moduli

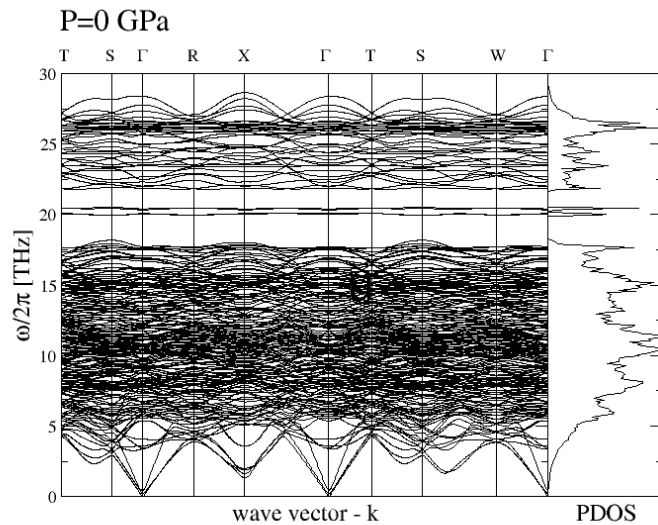
(Smyth et al. (2005))

Wadsl. II	H <sub>2</sub> O wt%	K (GPa)	K'
anhydrous <sup>a)</sup>	-	<b>167±10</b>	-
hydrous <sup>a)</sup>	<b>2.1</b>	<b>151±6</b>	<b>6±2.5</b>
	<b>2.8</b>	<b>145.6±2.8</b>	<b>6.1±0.7</b>
this study <sup>b)</sup>	-	<b>160.1</b>	<b>4.3</b>

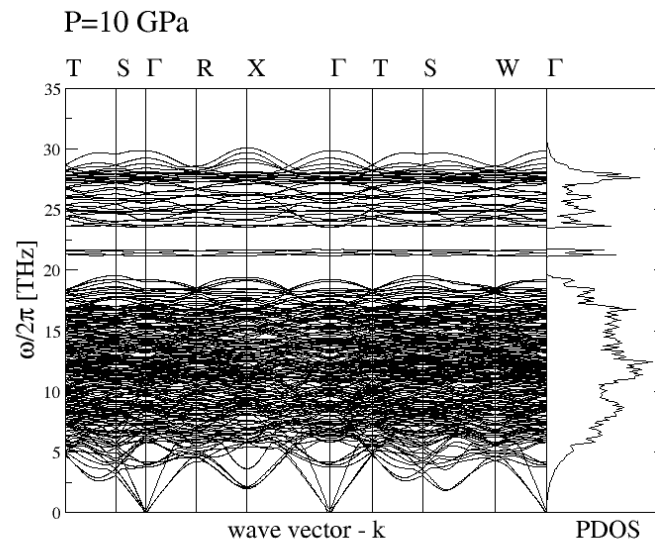
<sup>a)</sup> exper. Smyth et al., *Phys.Chem.Mineral* **31**,691 (2005)

<sup>b)</sup> calc. Tokár et al., *J.Phys.: Condens. Matter* **22**,145402 (2010)

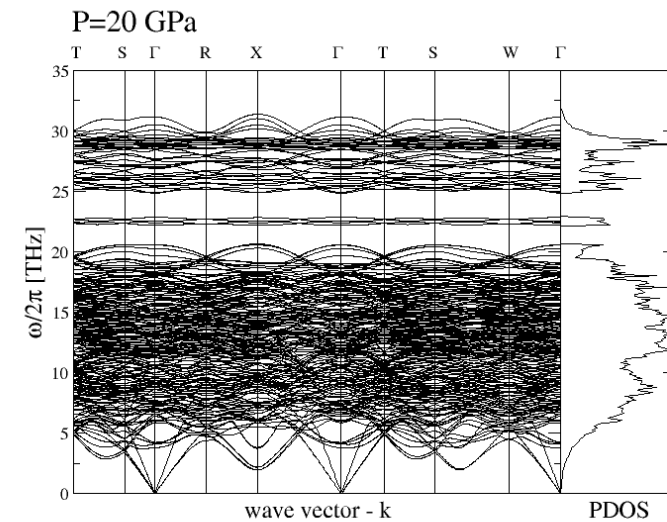
# Lattice stability of the Wadsleyite II at high pressure (calculation)



External pressure: P=(0-20 GPa)  
210 phonon vibration modes



Synthesis at ~17.5-18.5 GPa



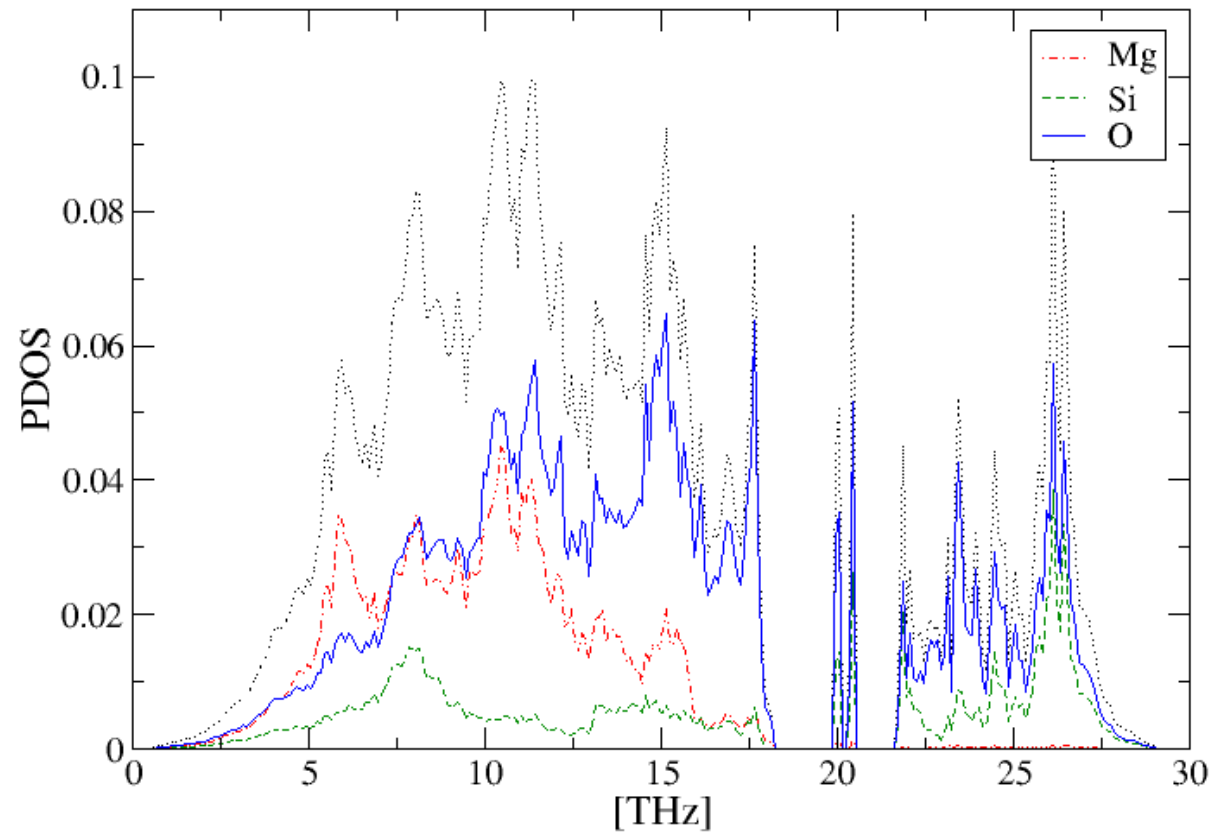
Phonon dispersion curves: **No soft phonon behavior**



**Conclusion:**  
Crystal of **wadsleyite II** has **mechanically stable structure**

# Lattice dynamics: Partial Phonon DOS of Wadsleyite II: $p=0$ GPa

## Wadsleyite II





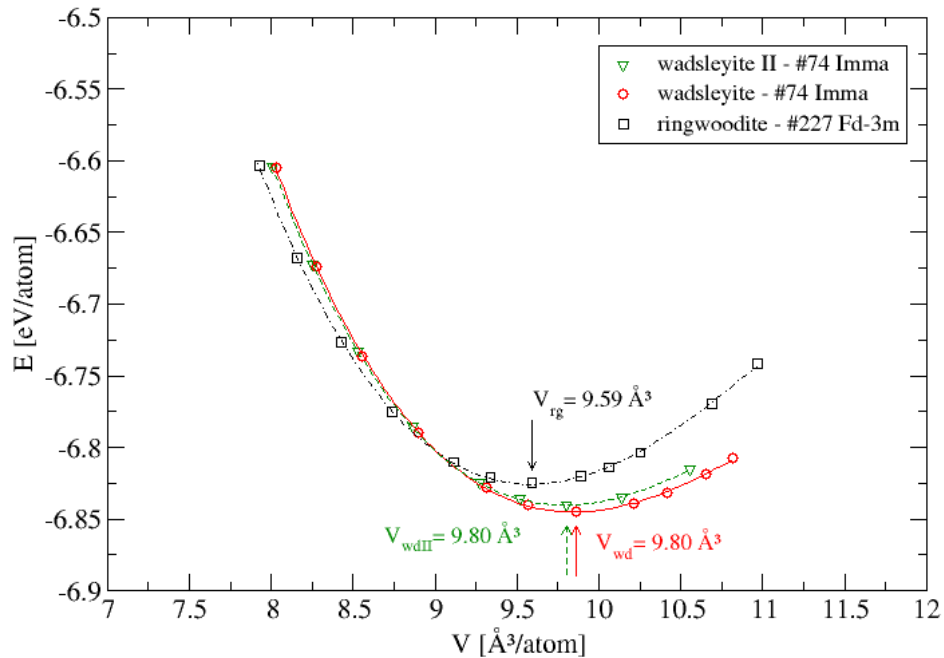
# Constraining of the complete pT diagram

Construction of thermodynamical model:

- **Lattice dynamics** calculated from *ab initio*
- **PDOS** of all considered  $\text{Mg}_2\text{SiO}_4$  phases: **wadsleyite**, ringwoodite, **wadsleyite II** + (periclase+perovskite)
- **QHA** approximation to parametrize thermodynamical functions of the phases and constrain coexistence boundaries

# QHA approximation used to derive pT diagram

DFT+GGA :: Static energy  $E_{stat}(V)$



Phase boundary:

Locus of points  $\{p, T\}$  with condition:

$$\mu_{\beta}(T, p) = \mu_{\gamma}(T, p) \implies \{p, T\}$$

Lattice dynamics:

$$PDOS\ g(\omega) \implies F_{phon}(T, V)$$

$$F_{phon}(T, V) = k_B T \int_0^{\infty} g(\omega) \ln\left(2sh\frac{\hbar\omega}{2k_B T}\right) d\omega$$

Free energy:

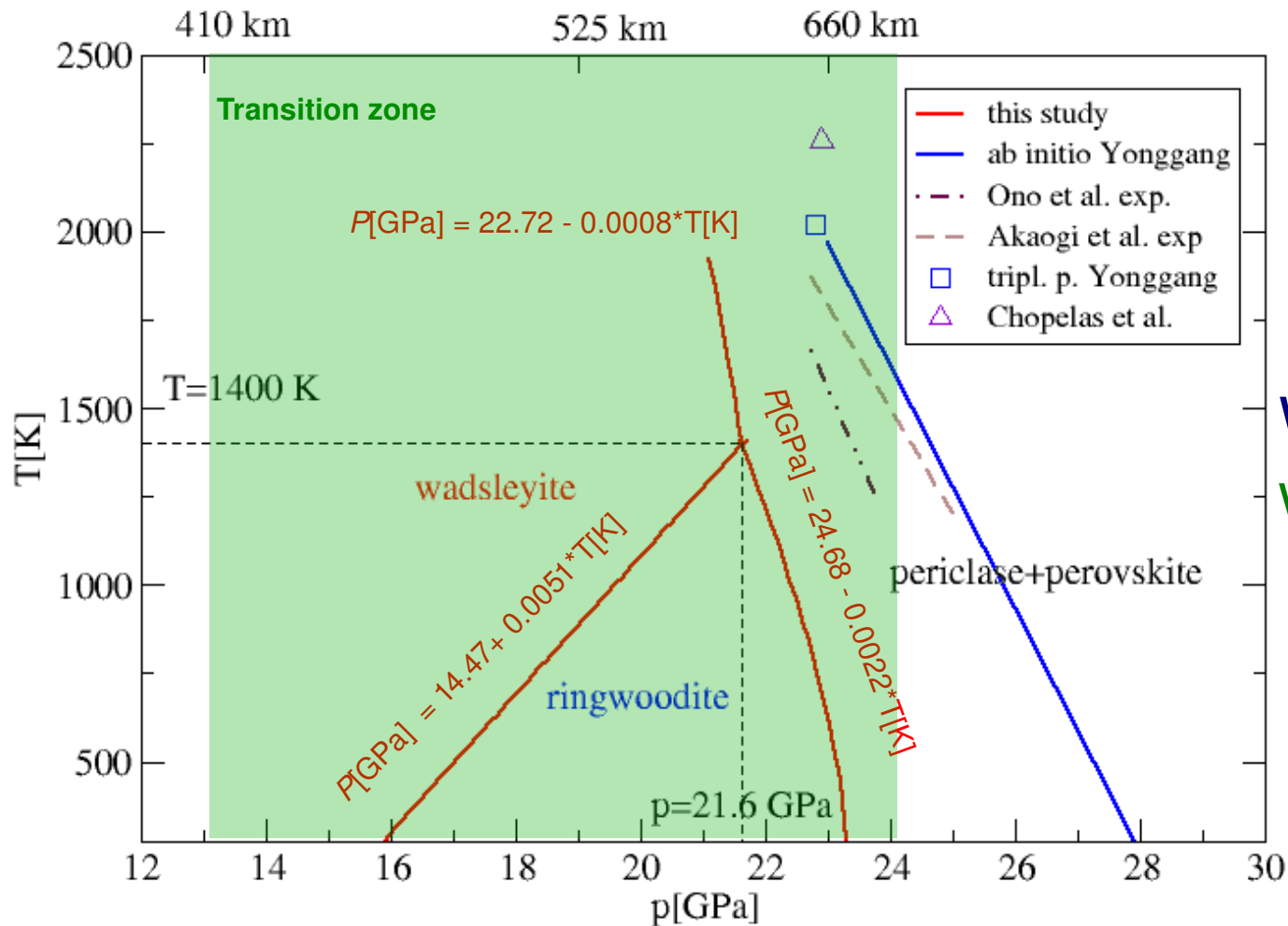
$$G_{phas}(T, p) = E_{stat}(V) + F_{phon}(T, V) + pV$$

Effective chemical potential:

$$\mu_{phas}(T, p) = G_{phas}(T, p) / N \quad :: \quad p = const.$$

$N$  – number of particles in the unit cell

# Complete $Mg_2SiO_4$ phase diagram and triple point at high pressure



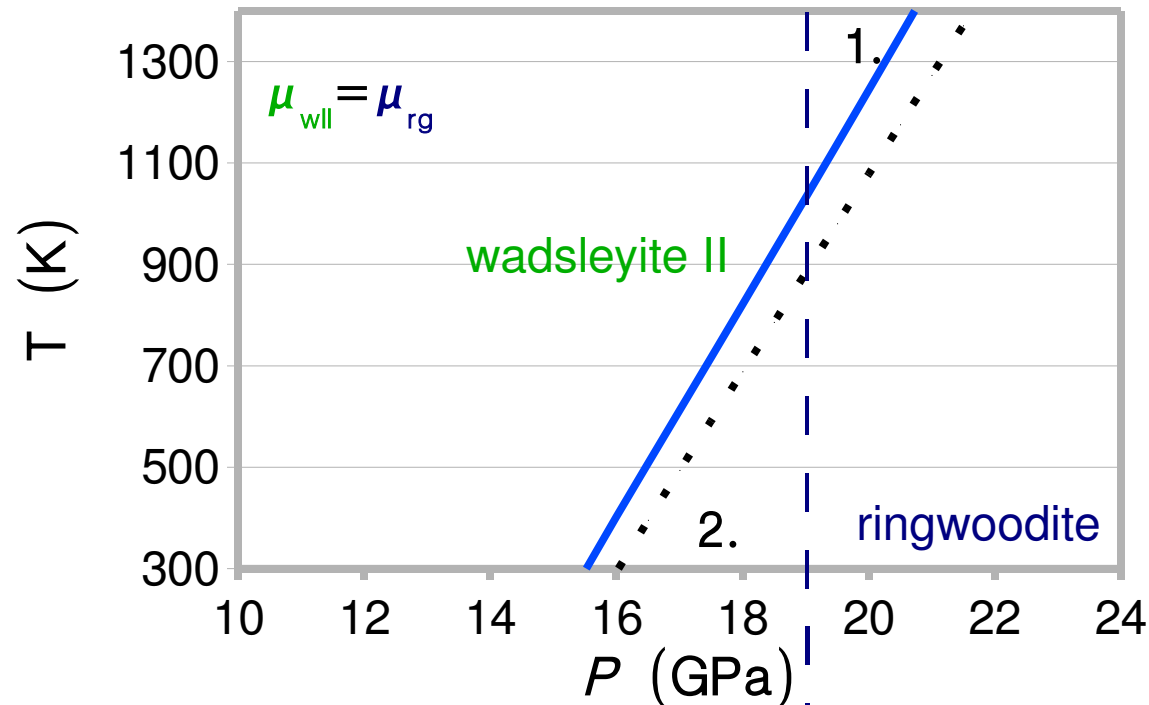
Cs=dp/dt

Where is  
wadsleyite II ??

**Triple point:** phases meet at  $p=21.6$  GPa,  $T=1400$  K

**Dissociation:**  $(\beta ; \gamma) - Mg_2SiO_4 \rightarrow \{ MgO(\text{periclase}) + MgSiO_3(\text{perovskite}) \}$

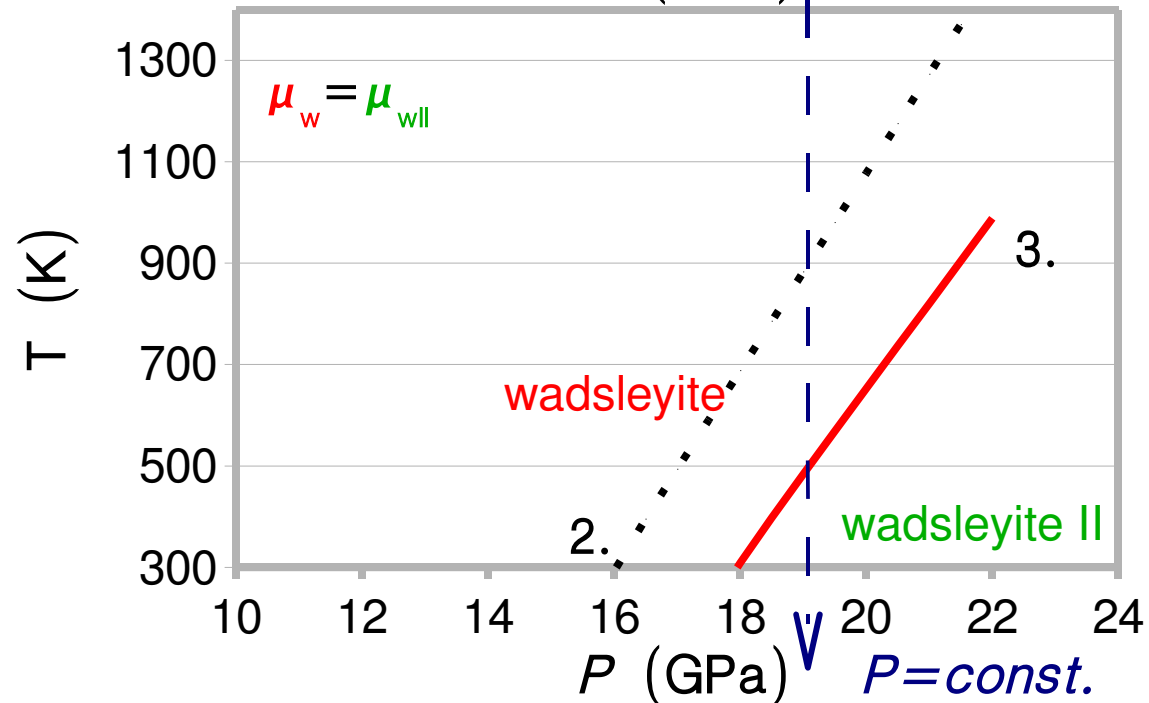
# Wadsleyite II phase relations to wadsleyite and ringwoodite (estimation)



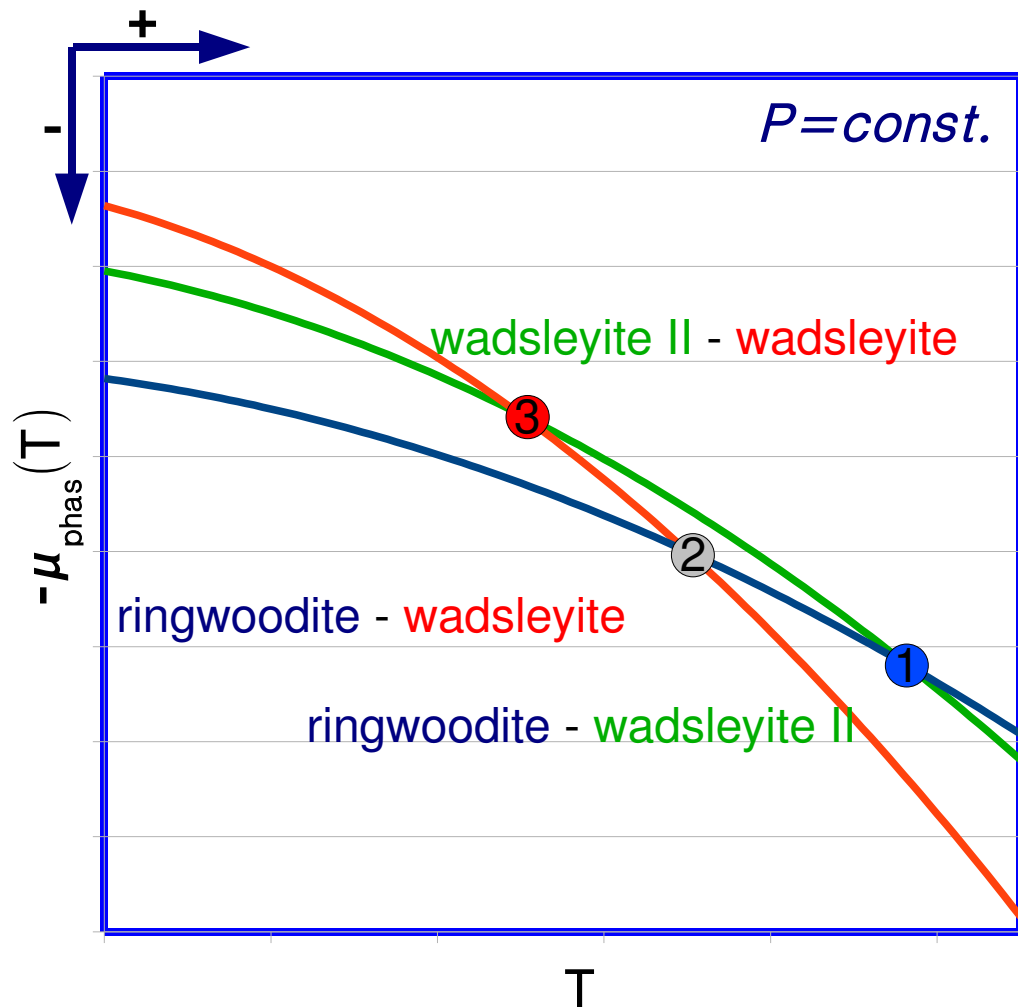
## Phase boundaries from QHA

### Clapeyron slopes:

1. — wadsleyite II-ringwoodite:  
 $P [\text{GPa}] = 14.13 + 0.0047 * T [\text{K}]$
2. - - - wadsleyite-ringwoodite:  
 $P [\text{GPa}] = 14.47 + 0.0051 * T [\text{K}]$
3. — wadsleyite-wadsleyite II:  
 $P [\text{GPa}] = 16.21 + 0.0058 * T [\text{K}]$



# Wadsleyite II phase thermodynamic stability in $Mg_2SiO_4$ system



## Thermodynamic relations:

$$(\mu_{wd} < \mu_{wdII}) \text{ OR } (\mu_{rg} < \mu_{wdII})$$

(wadsleyite & ringwoodite) preferred phases!!



~~wadsleyite II~~

Resolution interval:  $P \leq 22 \text{ GPa}$   $T = 0 - 2000 \text{ K}$

Conclusion: wadsleyite II is mechanically stable but not thermodynamically

## Conclusions

- Crystal lattice of **wadsleyite II** is mechanically stable up to 30 GPa
- **Wadsleyite II** is thermodynamically unstable in the magnesium-orthosilicate system and can be considered as an intermediate phase
- **Wadsleyite II** structure could be stabilized by Fe ions doping

Thank you.